4. [12 points]

a. [6 points] Determine if the following integral converges or diverges and circle the corresponding word. You must show all work and indicate any theorems you use. You do not need to calculate the value of the integral if it converges:

$$\int_4^{\infty} \frac{x}{3x^2 - \ln x} dx$$

Circle one: **Converges** **Diverges**

Solution: On the interval $[4, \infty)$,

$$\frac{x}{3x^2 - \ln x} \geq \frac{x}{3x^2} = \frac{1}{3x}.$$

By the p-test ($p = 1$), $\int_4^{\infty} \frac{1}{3x} dx$ diverges. Therefore, by the (Direct) Comparison Test, $\int_4^{\infty} \frac{x}{3x^2 - \ln x} dx$ diverges.

b. [6 points] Determine whether the following series converges or diverges and circle the appropriate answer. **Fully justify** your answer including using **proper notation** and showing mechanics of any tests you use:

$$\sum_{n=1}^{\infty} \frac{100^n}{n(99^n)}$$

Circle one: **Converges** **Diverges**

Solution: Let $a_n = \frac{100^n}{n(99)^n}$.
Method 1:

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{(100/99)^n}{n} \overset{\text{L'H}\frac{\infty}{\infty}}{=} \lim_{n\to\infty} \frac{\ln(100/99)(100/99)^n}{1} = \infty.$$

So, by the nth term test, the series $\sum_{n=1}^{\infty} a_n$ diverges.
Method 2: Using the ratio test, since $a_n \geq 0$,

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{100^{n+1}}{(n+1)(99)^{n+1}} \frac{n(99)^n}{100^n} = \lim_{n\to\infty} \frac{100n}{99(n+1)} = \frac{100}{99} > 1.$$

So, by the Ratio Test, the series $\sum_{n=1}^{\infty} a_n$ diverges.