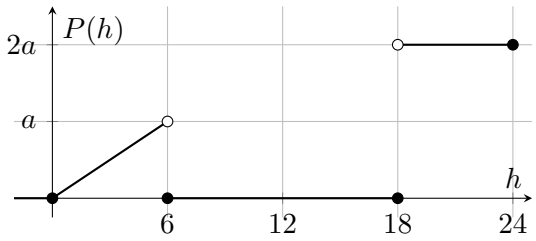
5. [10 points] George's mom's birthday party is in 24 hours and George still hasn't bought her a present. The mall near George is open for the next 6 hours, then closes for 12 hours, and then is open another 6 hours tomorrow before the party starts. George will search the mall until he finds the perfect present.

Below is a **partial** graph of $P(h)$, the probability density function (pdf) representing how long it will take George to find the perfect present in h hours. Assume $a > 0$ is some constant and $P(h) = 0$ for any $h \leq 0$.



a. [3 points] If the probability George finds the perfect present for his mom before the party starts is 1, what is the correct value of a in the graph above?

Solution: The area under the part of the graph that is visible is $\frac{1}{2}(6a) + 12a$. Setting this to be 1,

$$15a = 1$$
$$a = \frac{1}{15}$$

It is important to note for parts ***b.*** *and* ***c.*** *that only a* ***partial*** *graph of the function $P(h)$ is shown.*

b. [3 points] Now suppose $a = \frac{1}{20}$. What is the probability George will **not** find a present before the start of the party?

Solution: Now the area under the graph is still $15a$, but now $15a = \frac{15}{20} = \frac{3}{4}$. So, there is a 25% chance George will not find a present.

c. [4 points] In the case that $a = \frac{1}{20}$, finish the sentence to write a practical interpretation for the statement $P(26) = .02$:

There is approximately a 1% chance that....

Solution: There is approximately a 1% chance that George will take between 26 & 26.5 hours to find a present for his mom.
Note that $(26.5 - 26).02 = .01 = 1\%$. In particular, we could have used any interval around 26 that has length 0.5.