6. [14 points] Determine whether each of the following series converge conditionally, converges absolutely, or diverges and circle the appropriate answer. **Fully justify** your answer including using **proper notation** and showing mechanics of any tests you use.

a. [7 points]

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{6+\sqrt{n}}$$

Circle one: **Absolutely Converges** **Conditionally Converges** **Diverges**

Solution: First, we use the alternating series test to show the series itself converges: Let $a_n = \frac{1}{6+\sqrt{n}}$. It is easily verifiable that

$$\lim_{n\to\infty} a_n = 0,$$
$$a_n > 0 \text{ for } n \geq 1$$
$$a_n > a_{n+1} > 0.$$

So, by the Alternating Series Test, the series $\displaystyle\sum_{n=1}^{\infty}(-1)^n a_n$ converges.

Now, let's show that $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges: Note that for $n \geq 1$,

$$\frac{1}{6+\sqrt{n}} \geq \frac{1}{7\sqrt{n}}.$$

By the p-test for series($p = \frac{1}{2}$), $\sum_{n=1}^{\infty} \frac{1}{7\sqrt{n}}$ diverges. So, by the (Direct) Comparison Test $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges.

6. (continued) Here is a reproduction of the instructions for the problem:
Determine whether each of the following series converge conditionally, converges absolutely, or diverges and circle the appropriate answer. **Fully justify** your answer including using **proper notation** and showing mechanics of any tests you use.

b. [7 points]

$$\sum_{n=1}^{\infty} \frac{n^2 + 50n \sin 2n}{n^{7/2}}$$

Circle one: **Absolutely Converges** **Conditionally Converges** **Diverges**

Solution: Set $a_n = \dfrac{n^2 + 50n \sin 2n}{n^{7/2}}$. Note that a_n can be positive or negative, but does not alternate. We have for $n \geq 1$,

$$|a_n| \leq \frac{n^2 + 50n}{n^{7/2}} \leq \frac{51n^2}{n^{7/2}} = 51n^{-3/2}.$$

By the p-test for series ($p = 3/2$, $\sum_{n=1}^{\infty} \frac{51}{n^{3/2}}$ converges. So, by the (Direct) Comparison Test, $\sum_{n=1}^{\infty} |a_n|$ converges. So $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.