7. [8 points] Suppose $F$ is a nonnegative function defined for all real numbers $x$. Below are properties of $F$. Circle all that apply to $F$ based on the fact it has this property.
a. $[2$ points $] \int_{-\infty}^{\infty} F(x) d x=1$.
(A) $F$ could be a PDF.
(B) $F$ could be a CDF.
(C) $F$ is definitely not a PDF or a CDF.
b. [2 points] $\lim _{x \rightarrow \infty} F(x)=1$ and $F(2)<F(1)$.
(A) $F$ could be a PDF.
(B) $F$ could be a CDF.
(C) $F$ is definitely not a PDF or a CDF.
c. [2 points] $F^{\prime}(x)>0$ for $x \geq 0$.
(A) $F$ could be a PDF.
(B) $F$ could be a CDF.
(C) $F$ is definitely not a PDF or a CDF.
d. [2 points] $F(7)=2$.
(A) $F$ could be a PDF.
(B) $F$ could be a CDF.
(C) $F$ is definitely not a PDF or a CDF.
8. [9 points] The parts of this problem are unrelated.
a. [4 points] Let $\sum_{n=1}^{\infty} a_{n}$ be a geometric series with $a_{3}=54$ and $a_{6}=-2$. Write a general formula for $a_{n}$ :
Solution: Since $a_{n}$ are the terms of a geometric series, $a_{3}=a b^{3}$ and $a_{6}=a b^{6}$ for some $a, b$ that we want to solve for. To solve for $b$ :

$$
\frac{a_{6}}{a_{3}}=\frac{-2}{54}=-\frac{1}{27}=b^{3}
$$

So, $b=\frac{-1}{3}$. Now, to solve for $a$ :

$$
a_{3}=54=a\left(\frac{-1}{3}\right)^{3}=\frac{-a}{27} .
$$

So, $a=-27 * 54$. This gives a general formula for $a_{n}=-27 * 54\left(\frac{-1}{3}\right)^{n}$
b. [5 points] Let $b_{n}=\frac{n}{n+1}$ and $s_{n}=\sum_{i=1} b_{i}$. Circle all statements which are true.
(A) The sequence $b_{n}$ is bounded.
(D) The sequence $s_{n}$ is bounded.
(B) The sequence $b_{n}$ is monotone.
(E) The sequence $s_{n}$ is monotone.
(C) $\lim _{n \rightarrow \infty} b_{n}$ exists.
(F) $\lim _{n \rightarrow \infty} s_{n}$ exists.

