7. [8 points] Suppose F is a nonnegative function defined for all real numbers x. Below are properties of F. Circle **all** that apply to F based on the fact it has this property.

a. [2 points] $\int_{-\infty}^{\infty} F(x)dx = 1$.

[(A)] F could be a PDF.
(B) F could be a CDF.
(C) F is definitely not a PDF or a CDF.

b. [2 points] $\lim_{x\to\infty} F(x) = 1$ and $F(2) < F(1)$.

(A) F could be a PDF.
(B) F could be a CDF.
[(C)] F is definitely not a PDF or a CDF.

c. [2 points] $F'(x) > 0$ for $x \geq 0$.

(A) F could be a PDF.
[(B)] F could be a CDF.
(C) F is definitely not a PDF or a CDF.

d. [2 points] $F(7) = 2$.

[(A)] F could be a PDF.
(B) F could be a CDF.
(C) F is definitely not a PDF or a CDF.

8. [9 points] The parts of this problem are unrelated.

a. [4 points] Let $\sum_{n=1}^{\infty} a_n$ be a geometric series with $a_3 = 54$ and $a_6 = -2$. Write a general formula for a_n:

Solution: Since a_n are the terms of a geometric series, $a_3 = ab^3$ and $a_6 = ab^6$ for some a, b that we want to solve for. To solve for b:

$$\frac{a_6}{a_3} = \frac{-2}{54} = -\frac{1}{27} = b^3$$

So, $b = \frac{-1}{3}$. Now, to solve for a:

$$a_3 = 54 = a\left(\frac{-1}{3}\right)^3 = \frac{-a}{27}.$$

So, $a = -27 * 54$. This gives a general formula for $a_n = -27 * 54\left(\frac{-1}{3}\right)^n$

b. [5 points] Let $b_n = \frac{n}{n+1}$ and $s_n = \sum_{i=1}^{n} b_i$. Circle all statements which are true.

[(A)] The sequence b_n is bounded.
[(B)] The sequence b_n is monotone.
[(C)] $\lim_{n\to\infty} b_n$ exists.
(D) The sequence s_n is bounded.
[(E)] The sequence s_n is monotone.
(F) $\lim_{n\to\infty} s_n$ exists.