

3. [15 points] Carlos and Nancy are catching a train that leaves at 4pm. They leave their apartment for the train station at 12pm. The amount of time t (in hours) that elapses between the time they leave their apartment and the time they arrive at the train station is described by the following **probability density function** (pdf) $h(t)$:

$$h(t) = \begin{cases} 0 & t \leq 3 \\ a(t-3) & 3 < t \leq 4 \\ \frac{1}{4}e^{4-t} & 4 < t < \infty. \end{cases}$$

- a. [5 points] What is the probability they arrive late for their train (i.e., what is the probability they arrive at the train station after 4pm)? Be sure to show work for your calculations, and be sure to use proper notation.
- b. [4 points] Find the value of a so that $h(t)$ is a probability density function. Be sure to show work for any calculations.
- c. [3 points] Give a practical interpretation of the fact that $h(4.5) = 0.15$. Note that the output value has been rounded to the nearest hundredth.
- d. [3 points] Write an expression involving one or more integrals that gives the mean amount of time it takes Nancy and Carlos to travel to the train station. The letter h should not appear in your answer. You do not need to evaluate any integrals for this part.