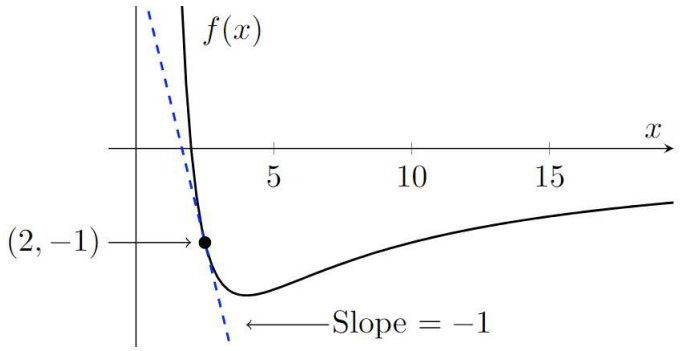
5. [12 points] The solid curve graphed below is part of the graph of a function $f(x)$ which has the following properties:

- $f(x)$ is twice differentiable on the interval $(0, \infty)$.
- $f(2) = -1$.
- For all $x \geq 10$, $f(x) < -\dfrac{5}{x}$.

The dashed line is the tangent line to $f(x)$ at $x = 2$, and its slope is -1.



a. [3 points] Compute $\lim\limits_{x \to 2} \left(\dfrac{f(x) + 1}{\cos\left(\frac{\pi}{2}x\right) + \frac{1}{2}x} \right)$

b. [3 points] Compute $\lim\limits_{x \to \infty} x[f(2 + x^{-1}) + 1]$

c. [6 points] Does the following improper integral converge or diverge? Fully justify your answer including using proper notation and showing mechanics of any tests you use.

$$\int_1^{\infty} (-f(x))\, dx.$$