1. [11 points]

a. [7 points] Determine the **radius** of convergence of the following power series:

$$\sum_{n=1}^{\infty} \frac{9^n (x-2)^{2n}}{n^2}$$

Be sure to show all of your work. Write your final answer in the space provided below.

Solution: We use the ratio test, with $a_n = \frac{9^n(x-2)^{2n}}{n^2}$. Then:

$$\begin{aligned} \frac{|a_{n+1}|}{|a_n|} &= \frac{9^{n+1}|x-2|^{2n+2}n^2}{(n+1)^2 9^n |x-2|^{2n}} \\ &= 9|x-2|^2 \frac{n^2}{(n+1)^2} \\ &\to 9|x-2|^2 \quad \text{as } n \to \infty. \end{aligned}$$

This is less than 1 exactly when $9|x-2|^2 < 1$, or in other words $|x-2| < 1/3$. So the radius of convergence is $1/3$.

b. [4 points] Suppose that the power series

$$\sum_{n=1}^{\infty} a_n (x-5)^n$$

Answer: $1/3$.

converges when $x = 10$ and diverges when $x = -1$. At which of the following x-values must the series converge? Circle your answers. You do not need to show any work for this problem.

-5 0 **2** **5** 11 12

Solution: From the information given, the radius of convergence is at least 5 and at most 6. Hence it definitely converges for $|x-5|<5$, but we don't know if it converges when $|x-5|\geq 5$. So it definitely converges at $x=2,5$.