10. [14 points] Determine if the following series converge or diverge. Circle your final answer choice for each. Fully justify your answer including using proper notation and showing mechanics of any tests you use.

a. [7 points] $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n}{n^2+4n-1}$

Circle one: $\boxed{\textbf{Converges}}$ **Diverges**

Solution: With $a_n = \frac{n}{n^2+4n-1}$, we see that $a_n > a_{n+1}$ for all $n \geq 1$. Furthermore, $\lim\limits_{n\to\infty} a_n = 0$. By the alternating series test, our original series converges.

b. [7 points] $\displaystyle\sum_{n=1}^{\infty} \frac{2n-1}{n^2+n+2}$

Circle one: **Converges** $\boxed{\textbf{Diverges}}$

Solution: By leading term analysis, this series has terms reminiscent of $\frac{2n}{n^2} = \frac{2}{n}$. Since $\sum_{n=1}^{\infty} \frac{2}{n}$ diverges by the p-test with $p = 1$, we expect divergence. To justify this we use LCT. Let $b_n = \frac{2}{n}$ and let $a_n = \frac{2n-1}{n^2+n+2}$. Then

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{2n^2-n}{2n^2+2n+4} = 1.$$

Since $0 < 1 < \infty$, LCT implies that the given series and our comparison series have the same behavior. Therefore our original series diverges.