2. [8 points] Suppose that a_n, b_n, and c_n are sequences with the following properties:

- The sequence a_n is bounded
- The series $\sum_{n=1}^{\infty} b_n$ converges absolutely
- $\frac{1}{n^2+1} \leq c_n \leq \frac{1}{n}$ for all $n \geq 1$

Determine whether the following statements are **always**, **sometimes**, or **never** true, and circle the appropriate answer for each part. No justification is necessary.

a. [2 points] The sequence b_n converges to 0.

Circle one: [✓ **Always**] **Sometimes** **Never**

Solution: This is always true by the nth term test for divergence.

b. [2 points] $\sum_{n=1}^{\infty} \frac{c_n}{n}$ diverges.

Circle one: **Always** **Sometimes** [✓ **Never**]

Solution: This is never true by the comparison test, with comparison series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

c. [2 points] The sequence a_n converges.

Circle one: **Always** [✓ **Sometimes**] **Never**

Solution: This is sometimes true. For example, the sequence $a_n = 0$ for all $n \geq 1$ is bounded and converges; on the other hand, the sequence $a_n = (-1)^n$ for $n \geq 1$ is bounded and does not converge.

d. [2 points] The series $\sum_{n=1}^{\infty} \frac{1}{n^3 c_n}$ converges.

Circle one: **Always** [✓ **Sometimes**] **Never**

Solution: This is sometimes true. By multiplying through by n^3 and then taking reciprocals of the inequality that c_n satisfies, we obtain

$$\frac{1}{n^2} = \frac{n}{n^3} \leq \frac{1}{n^3 c_n} \leq \frac{n^2+1}{n^3}.$$

If $\frac{1}{n^3 c_n} = \frac{1}{n^2}$, then this is an example where the corresponding series converges; on the other hand, if $\frac{1}{n^3 c_n} = \frac{n^2+1}{n^3}$, this is an example where the corresponding series diverges.