

4. [9 points] Consider the following sequences, all defined for  $n = 1, 2, 3, \dots$

$$a_n = \int_1^n \frac{1}{x^2} dx$$

$$b_n = 1 + \frac{(-1)^n(n+1)}{n^2}$$

$$c_n = \sum_{k=1}^n \frac{1}{(k+1)^{0.5}}$$

- a. [3 points] Which sequences are monotone? No justification is required for this part of the problem. Circle your final answer(s) below.

Circle your answers:   $a_n$    $b_n$    $c_n$   NONE

- b. [3 points] Which sequences are convergent? No justification is required for this part of the problem. Circle your final answer(s) below.

Circle your answers:   $a_n$    $b_n$    $c_n$   NONE

- c. [3 points] Does the series  $\sum_{n=1}^{\infty} a_n$  converge? Justify your answer.

*Solution:* Note that

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \int_1^n \frac{1}{x^2} dx = \int_1^{\infty} \frac{1}{x^2} dx.$$

This integral converges to a nonzero number. By the  $n$ th term test for divergence, the series  $\sum_{n=1}^{\infty} a_n$  diverges.