7. [7 points] Determine whether the following improper integral converges or diverges. Circle your final answer choice. Fully justify your answer including using proper notation and showing mechanics of any tests you use.

$$\int_1^\infty \frac{t^2 - \ln(t)}{t^4 + 8t + 10}\, dt.$$

Circle one: **Converges** **Diverges**

Solution: The numerator is dominated by t^2, and the denominator is dominated by t^4, so the integrand has the same behavior (for large t) as $\frac{t^2}{t^4} = \frac{1}{t^2}$, whose integral on the interval $[1, \infty)$ converges. Therefore we expect that this improper integral converges. To show this, first note that since $\ln(t) \geq 0$ for $t \geq 1$, we have

$$t^2 - \ln(t) \leq t^2.$$

Also, since $8t + 10 \geq 0$ for $t \geq 1$, we have

$$t^4 + 8t + 10 \geq t^4.$$

Therefore

$$\frac{t^2 - \ln(t)}{t^4 + 8t + 10} \leq \frac{t^2}{t^4} = \frac{1}{t^2}.$$

Now, $\int_1^\infty \frac{1}{t^2}\, dt$ converges by the p-test with $p = 2$. Hence, by the comparison test, our integral converges as well.

8. [5 points] Fully evaluate the following integral:

$$\int x \ln x \, dx$$

You do not need to simplify your answer.