4. [12 points] The population of fungus-eating insects Megan introduced to stop her mushroom infestation has grown out of control! She decides to spray pesticide to slow their growth. Over the first half of each day, the population of insects increases by 100 percent. Throughout the last half of the day, Megan sprays her garden with insecticide and wipes out exactly k insects. The insects only multiply during the first half of a day, and Megan is only awake to spray the colony during the last half of a day.

a. [5 points] Let B_n denote the number of insects in the colony at the start of the nth day. At the start of the first day, there were 133 insects in the colony (so $B_1 = 133$). Find expressions for the values of B_2, B_3, and B_4. Your answers will involve k but should not involve the letter B. You do not need to simplify your expressions.

$B_2 =$ _____

$B_3 =$ _____

$B_4 =$ _____

b. [5 points] Find a closed-form expression for B_n. Closed form means your answer should not include ellipses or sigma notation, and should NOT be recursive. Your answer will involve k. You do not need to simplify your expression.

Answer: _____

c. [2 points] Megan can affect the value of k by adjusting the amount of pesticide she sprays. What value must k be so that the population of insects at the start of the nth day is the same for all $n = 1, 2, \ldots$? (Hint: Use your expressions from part **a**.)

Answer: _____