8. [12 points]

a. [7 points] Determine the **radius** of convergence for the following power series. Show all of your work. You do not need to find the interval of convergence.

$$\sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{9^n (n!)^2} x^{3n}$$

Answer: ______________________

b. [5 points] No justification is needed for the remainder of this problem. Suppose that the following is true about the sequence C_n which is defined for $n \geq 0$:

- C_n is monotone decreasing and converges to 0.
- $\sum_{n=0}^{\infty} C_n$ diverges.
- The power series $\sum_{n=0}^{\infty} \frac{(-1)^n C_n}{6^n}(x-5)^n$ has radius of convergence 6.

What is the center of the interval of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n C_n}{6^n}(x-5)^n$?

Answer: ______________________

What are the endpoints of the interval of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n C_n}{6^n}(x-5)^n$?

Answer: Left endpoint at $c =$ ______________

Right endpoint at $d =$ ______________

Let c and d be the left and right endpoints of the interval of convergence you found above. Which of the following could be the interval of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n C_n}{6^n}(x-5)^n$? Circle **all** correct answers.

(c, d) $\qquad$ $(c, d]$ $\qquad$ $[c, d)$ $\qquad$ $[c, d]$