9. [12 points] For the following questions, determine if the statement is ALWAYS true, SOMETIMES true, or NEVER true, and circle the corresponding answer. Justification is not required.

a. [2 points] If the series $\sum_{n=1}^{\infty}(-1)^n a_n$ diverges, then the series $\sum_{n=1}^{\infty} a_n$ also diverges.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

b. [2 points] If b_n is a sequence of positive numbers which satisfy $\lim_{n\to\infty} \frac{1}{n^3 b_n} = 12$, then $\sum_{n=1}^{\infty} b_n$ converges.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

c. [2 points] If $f(x)$ is a continuous function so that $\int_0^{\infty} f(x)\,\mathrm{d}x$ converges, then $\int_{10}^{\infty} \left(f(x) + \frac{1}{x^5}\right)\,\mathrm{d}x$ converges too.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

d. [2 points] If $\sum_{n=0}^{\infty} d_n = \frac{1}{1-0.3}$, then $d_n = (0.3)^n$ for all $n \geq 0$.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

e. [2 points] The function given by

$$g(x) = \begin{cases} x^3, & -1 \leq x \leq \sqrt[4]{5}, \\ 0, & \text{otherwise}, \end{cases}$$

is a probability density function.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

f. [2 points] If s_n is a decreasing sequence of positive numbers which converges, then $\sum_{n=1}^{\infty} s_n$ converges too.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**