3. [12 points] Let $p(x)$ be a **probability density function** (pdf) such that

$$p(x) = \begin{cases} 1/10, & -3 \le x < 1, \\ 3/5, & 1 \le x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

a. [7 points] Find the cumulative distribution function $P(x)$ corresponding to $p(x)$.

Solution: The continuous function $P(x)$ should approach 0 as $x \to -\infty$, and should be an antiderivative of $p(x)$. We see that $P(x)$ is piecewise linear, and so we use point-slope form for each piece. Note that $P(-3) = 0$ and that, since $\int_{-3}^{1} p(x)\, \mathrm{d}x = \frac{2}{5}$, we must have $P(1) = \frac{2}{5}$. We obtain:

$$\textbf{Answer: } P(x) = \begin{cases} 0, & x < -3 \\ \frac{1}{10}(x+3), & -3 \le x < 1, \\ \frac{3}{5}(x-1) + \frac{2}{5}, & 1 \le x < 2, \\ 1, & x \ge 2 \end{cases}$$

b. [5 points] Find the mean value of x. Show all your work.

Solution: The mean value is given by:

$$\begin{aligned} \int_{-\infty}^{\infty} xp(x)\, \mathrm{d}x &= \int_{-3}^{1} \frac{1}{10} x\, \mathrm{d}x + \int_{1}^{2} \frac{3}{5} x\, \mathrm{d}x \\ &= \frac{1}{20} x^2 \Big|_{-3}^{1} + \frac{3}{10} x^2 \Big|_{1}^{2} \\ &= \frac{1}{20}(1-9) + \frac{3}{10}(4-1) \\ &= \frac{9}{10} - \frac{4}{10} = \frac{1}{2}. \end{aligned}$$

Answer: 0.5