**4.** [12 points] The population of fungus-eating insects Megan introduced to stop her mushroom infestation has grown out of control! She decides to spray pesticide to slow their growth.
Over the first half of each day, the population of insects increases by 100 percent. Throughout the last half of the day, Megan sprays her garden with insecticide and wipes out exactly $k$ insects. The insects only multiply during the first half of a day, and Megan is only awake to spray the colony during the last half of a day.

**a.** [5 points] Let $B_n$ denote the number of insects in the colony at the start of the $n$th day. At the start of the first day, there were 133 insects in the colony (so $B_1 = 133$). Find expressions for the values of $B_2$, $B_3$, and $B_4$. Your answers will involve $k$ but should not involve the letter $B$. You do not need to simplify your expressions.

$B_2 =$ $2 \cdot 133 - k$

$B_3 =$ $2^2 \cdot 133 - k - 2k$

$B_4 =$ $2^3 \cdot 133 - k - 2k - 4k$

**b.** [5 points] Find a closed-form expression for $B_n$. Closed form means your answer should not include ellipses or sigma notation, and should NOT be recursive. Your answer will involve $k$. You do not need to simplify your expression.

*Solution:* Continuing the pattern above, we see $B_n = 2^{n-1} \cdot 133 - k - 2k - \ldots - 2^{n-2}k$. Aside from the first term in this expression, this is a geometric series with initial term $-k$, common ratio 2, and with $n-1$ terms. Using our formula for a finite geometric series, we obtain:

$$B_n = 2^{n-1} \cdot 133 - k\left(\frac{1-2^{n-1}}{1-2}\right) = 2^{n-1} \cdot 133 - k(2^{n-1} - 1).$$

**Answer:** $2^{n-1} \cdot 133 - k(2^{n-1} - 1)$

**c.** [2 points] Megan can affect the value of $k$ by adjusting the amount of pesticide she sprays. What value must $k$ be so that the population of insects at the start of the $n$th day is the same for all $n = 1, 2, \ldots$? (Hint: Use your expressions from part **a**.)

*Solution:* We set $B_1 = B_2$ to get $133 = 2 \cdot 133 - k$. Rearranging gives $k = 133$.

**Answer:** 133