1. [10 points] Compute the **exact value** of each of the following. If a value **diverges, or otherwise does not exist**, write DNE. If there is **not enough information** to determine a given value, write NEI. You do not need to justify or simplify your answers.

a. [2 points] Find the value of p so that $\int_0^{10} \frac{1}{x^{2p}}\,dx$ and $\int_3^{\infty} \frac{1}{x^{2p}}\,dx$ both diverge.

Solution: For the first integral to diverge, we must have $2p \geq 1$. For the second integral to diverge we must have $2p \leq 1$. Therefore $2p = 1$, so $p = \frac{1}{2}$.

Answer: $p =$ 0.5

b. [2 points] Recall that a normal distribution has a probability density function (pdf) of the form

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2},$$

where μ is the mean of the distribution and σ is the standard deviation, with $\sigma > 0$. Find the exact value of

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-(x-5)^2/18}\,dx.$$

Solution: Notice that for a normal distributions with mean 5 and standard deviation 3, the pdf is

$$\frac{1}{3\sqrt{2\pi}} e^{-(x-5)^2/18},$$

and since the area under a pdf is 1, we must have

$$\int_{-\infty}^{\infty} \frac{1}{3\sqrt{2\pi}} e^{-(x-5)^2/18}\,dx = 1.$$

Answer: 3

c. [2 points] Evaluate $\int_{-17}^{17} \frac{1}{x^2}\,dx$.

Solution: This integral diverges (by the p-test with $p = 2$).

Answer: **DNE**

d. [2 points] Find the exact value of the infinite sum $5 + \frac{10}{3} + \frac{20}{9} + \frac{40}{27} + \cdots$.

Solution: This is an infinite geometric series with initial term 5 and common ratio $\frac{2}{3}$, so the infinite sum is $\frac{5}{1-\frac{2}{3}} = 15$.

Answer: 15

e. [2 points] Let $q(x)$ be a probability density function (pdf) for a statistic with mean value 5. Find the exact value of $\int_{-\infty}^{\infty} (1+x)q(x)\,dx$.

Solution: We have $\int_{-\infty}^{\infty} q(x)\,dx = 1$, and $\int_{-\infty}^{\infty} xq(x)\,dx = 5$, so $\int_{-\infty}^{\infty} (1+x)q(x)\,dx = \int_{-\infty}^{\infty} q(x)\,dx + \int_{-\infty}^{\infty} xq(x)\,dx = 6$.

Answer: 6

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