11. [12 points]

a. [7 points] Determine the **radius** of convergence for the following power series. Show all of your work. You do not need to find the interval of convergence.

$$\sum_{n=1}^{\infty} \frac{(2n)!}{10^n (n!)(n+1)!} x^{2n}$$

Solution: We use the ratio test:

$$\begin{aligned}
\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| &= \lim_{n\to\infty} \frac{(2(n+1))!|x^{2(n+1)}|}{10^{n+1}(n+1)!(n+2)!} \cdot \frac{10^n(n!)(n+1)!}{(2n)!|x^{2n}|} \\
&= \lim_{n\to\infty} \frac{(2n+2)(2n+1)}{10(n+1)(n+2)}|x^2| \\
&= \lim_{n\to\infty} \frac{4n^2}{10n^2}|x^2| \\
&= \frac{4}{10}|x^2|
\end{aligned}$$

The ratio test tells us the power series converges when this value is smaller than 1, i.e. $\frac{4}{10}\left|x^2\right| < 1$. Rearranging, we see that this implies $|x|^2 < \left(\frac{10}{4}\right)$, which tells us that the radius of convergence is $\sqrt{\frac{10}{4}}$.

Answer: $\frac{\sqrt{10}}{2}$

b. [5 points] No justification is needed for the remainder of this problem. Suppose that the following is true about the sequence C_n which is defined for $n \geq 0$:

- C_n is a monotone decreasing sequence of positive numbers which converges to 0.
- $\lim_{n\to\infty} \frac{C_n}{1/n} = 28$.
- The power series $\sum_{n=0}^{\infty} \frac{C_n}{4^n}(x-16)^n$ has radius of convergence 4.

What is the center of the interval of convergence of $\sum_{n=0}^{\infty} \frac{C_n}{4^n}(x-16)^n$?

Answer: 16

What are the endpoints of the interval of convergence of $\sum_{n=0}^{\infty} \frac{C_n}{4^n}(x-16)^n$?

Answer: Left endpoint at $c =$ 12

Right endpoint at $d =$ 20

Let c and d be the left and right endpoints of the interval of convergence you found above. Which of the following could be the interval of convergence of $\sum_{n=0}^{\infty} \frac{C_n}{4^n}(x-16)^n$? Circle **all** correct answers.

(c, d) $(c, d]$ $\boxed{[c, d)}$ $[c, d]$