2. [5 points] Compute the following limit. Fully justify your answer including using **proper limit notation**.

$$\lim_{x\to\infty} 7x\ln\left(1+\frac{6}{x}\right)$$

Solution: We re-write the limit so that we may apply L'Hospital's Rule:

$$\begin{aligned}
\lim_{x\to\infty} 7x\ln\left(1+\frac{6}{x}\right) &= \lim_{x\to\infty}\frac{7\ln\left(1+\frac{6}{x}\right)}{1/x}\\
&\overset{\text{LH}}{=} 7\lim_{x\to\infty}\frac{\left(\frac{1}{1+\frac{6}{x}}\right)\left(-\frac{6}{x^2}\right)}{-1/x^2}\\
&= 7\lim_{x\to\infty}\frac{6x^2}{(x^2)\left(1+\frac{6}{x}\right)}\\
&= 7\lim_{x\to\infty}\frac{6x^2}{x^2+6x}\\
&= 7(6)\\
&= 42
\end{aligned}$$

Answer: $\lim_{x\to\infty} 7x\ln\left(1+\frac{6}{x}\right) =$ 42

3. [7 points] **Compute** the value of the following improper integral if it converges. If it does not converge, use a **direct computation** of the integral to show its divergence. Be sure to show your full computation, and be sure to use **proper notation**.

$$\int_2^{10}\frac{1}{(t-2)^{1/3}}\,\mathrm{d}t$$

Solution:

$$\begin{aligned}
\int_2^{10}\frac{1}{(t-2)^{1/3}}\,\mathrm{d}t &= \lim_{a\to 2^+}\int_a^{10}\frac{1}{(t-2)^{1/3}}\,\mathrm{d}t\\
&= \lim_{a\to 2^+}\left(\frac{3}{2}(t-2)^{2/3}\right)\Bigg|_a^{10}\\
&= \frac{3}{2}(8)^{2/3} - \lim_{a\to 2^+}\frac{3}{2}(a-2)^{2/3}\\
&= 6
\end{aligned}$$

Circle one: **Diverges** $\boxed{\textbf{Converges to } 6}$