4. [7 points] Determine whether the following improper integral converges or diverges and circle the corresponding word. **Fully justify** your answer including using **proper notation** and showing mechanics of any tests you use. You do not need to calculate the value of the integral if it converges.

$$\int_1^\infty \frac{12 + 5\sin(x)}{x^{1/4} + x^{5/4}}\, \mathrm{d}x$$

Circle one: **Converges** **Diverges**

Justification:

Solution: On the interval $1 \le x \le \infty$, we have $\dfrac{12 + 5\sin(x)}{x^{1/4} + x^{5/4}} \le \dfrac{17}{x^{5/4}}$, and $\displaystyle\int_1^\infty \frac{17}{x^{5/4}}\, \mathrm{d}x$ converges by the p-test with $p = \dfrac{5}{4}$. Therefore, by the (Direct) Comparison Test, $\displaystyle\int_1^\infty \frac{12 + 5\sin(x)}{x^{1/4} + x^{5/4}}\, \mathrm{d}x$ converges.