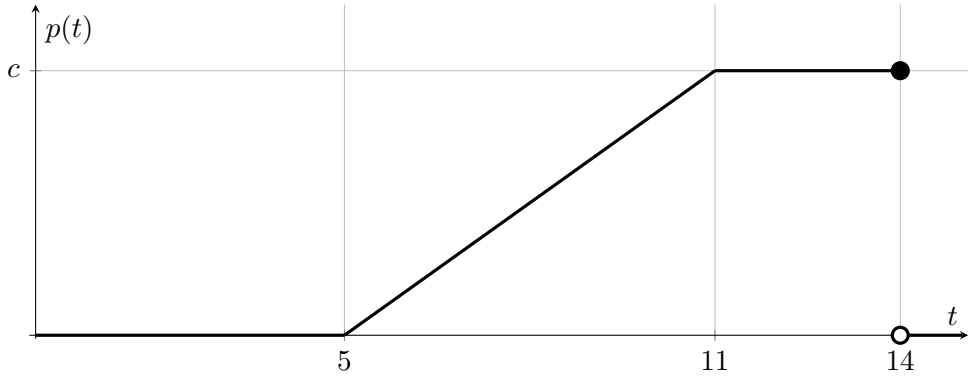
5. [9 points] Littorina the snail competes in a weekly race. The probability density function (pdf), $p(t)$ which describes the time in minutes it takes Littorina to finish the weekly race is depicted below. Note that $p(t)$ is piecewise linear, and that $p(t) = 0$ for $t < 5$ and $t > 14$.



a. [3 points] Find the value of c which makes $p(t)$ a probability density function.

Solution: For $p(t)$ to be a pdf, the area underneath the graph must be equal to 1. Therefore, $3c + 3c = 1$, and so $c = \frac{1}{6}$.

Answer: $c = \frac{1}{6}$

b. [6 points] Find the function $P(t)$ which describes the probability that Littorina completes the weekly race in t minutes or less. Your formula should not contain any integral signs, but may include the letter c.

Solution:
The function $P(t)$ must be a continuous antiderivative of $p(t)$ which satisfies $\lim_{t\to-\infty} P(t) = 0$ and $\lim_{t\to\infty} P(t) = 1$. The slope of $p(t)$ on $5 \le t \le 14$ is $\frac{c}{6}$, and so we have:

$$P(t) = \begin{cases} 0, & t < 5 \\ \frac{c}{12}(t-5)^2, & 5 \le t < 11, \\ \frac{1}{2} + c(t-11), & 11 \le t < 14, \\ 1, & t \ge 14 \end{cases}$$

Plugging in $c = \frac{1}{6}$,

Answer: $P(t) = \begin{cases} 0, & t < 5 \\ \frac{(t-5)^2}{72}, & 5 \le t < 11, \\ \frac{1}{2} + \frac{1}{6}(t-11), & 11 \le t < 14, \\ 1, & t \ge 14 \end{cases}$