8. [10 points] Determine if the following series converges absolutely, converges conditionally, or diverges, and circle the corresponding option. **Fully justify** your answer including using **proper notation** and showing mechanics of any tests you use.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^{0.3}}$$

Circle one: **Converges Absolutely** **Converges Conditionally** **Diverges**

Justification:

Solution: Note that the series is alternating. Let $a_n = \frac{1}{n\,(\ln(n))^{0.3}}$. Then for all n, $0 < a_{n+1} < a_n$, and we also have $\lim_{n\to\infty} a_n = 0$. Therefore, by the alternating series test, $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\,(\ln(n))^{0.3}}$ converges.

Now consider the series $\sum_{n=2}^{\infty} \left|\frac{(-1)^n}{n\,(\ln(n))^{0.3}}\right| = \sum_{n=2}^{\infty} \frac{1}{n\,(\ln(n))^{0.3}}$.

Let $f(x) = \frac{1}{x\,(\ln(x))^{0.3}}$. Then for $x \geq 2$, $f(x)$ is positive and decreasing. We have:

$$\begin{aligned}
\int_3^{\infty} \frac{1}{x\,(\ln(x))^{0.3}}\,\mathrm{d}x &= \lim_{b\to\infty} \int_2^{b} \frac{1}{x\,(\ln(x))^{0.3}}\,\mathrm{d}x \\
&= \lim_{b\to\infty} \int_{\ln(2)}^{\ln(b)} \frac{1}{u^{0.3}}\,\mathrm{d}u \\
&= \lim_{b\to\infty} \frac{1}{0.7} u^{0.7}\Big|_{\ln(2)}^{\ln(b)} \\
&= \lim_{b\to\infty} \frac{1}{0.7}\left((\ln(b))^{0.7} - (\ln(2))^{0.7}\right) = \infty.
\end{aligned}$$

Therefore, $\int_2^{\infty} \frac{1}{x\,(\ln(x))^{0.3}}\,\mathrm{d}x$ diverges, and so by the Integral test, $\sum_{n=2}^{\infty} \frac{1}{n\,(\ln(n))^{0.3}}$ diverges too.

Therefore $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\,(\ln(n))^{0.3}}$ converges conditionally.