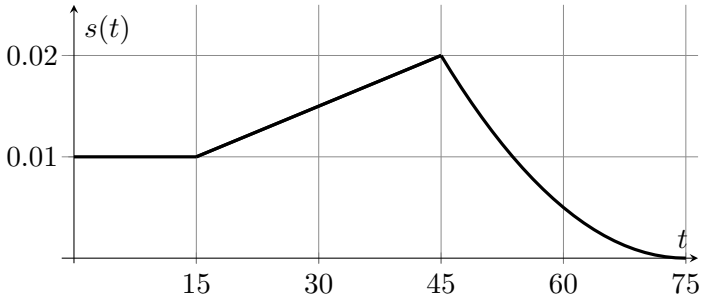
1. [9 points] Steph times their study sessions. Let t be the length in minutes, and let $s(t)$ be the probability density function for t. A **portion** of the graph of $s(t)$ is shown below. Also note that $s(t)$:

- is continuous;
- is constant for $0 \le t \le 15$;
- is linear for $15 \le t \le 45$;
- is concave up for $45 \le t \le 75$.



a. [3 points] Find the proportion of Steph's study sessions that last between 15 minutes and 45 minutes. You do not need to simplify your answer but you should show your work.

Solution: The proportion is equal to

$$\int_{15}^{45} s(t)\ dt = \frac{1}{2} \cdot 30 \cdot (0.01 + 0.02) = 0.45.$$

Answer: 0.45

b. [3 points] Which of the statements below is best supported by the expression $s(55) \approx 0.01$? Circle the **one** best statement. No justification is required.

i. Approximately 0.01% of sessions last *exactly* 55 minutes.

ii. Approximately 1% of sessions last between 54 and 56 minutes.

iii. Approximately 2% of sessions last between 54 and 56 minutes.

iv. Approximately 10% of sessions last between 55 and 56 minutes.

v. Approximately 55% of sessions last around 0.01 minutes.

c. [3 points] Do any sessions last longer than 75 minutes? Give a brief explanation of your answer.

Circle one: YES NO NOT ENOUGH INFORMATION

Explanation:

Solution: We can compute $\int_0^{45} s(t)\ dt = 0.6$, and since $s(t)$ is concave up for $45 \le t \le 75$, we have $\int_{45}^{75} s(t)\ dt \le 0.3$ (equality would hold if $s(t)$ was linear for $45 \le t \le 75$). Therefore, $\int_0^{75} s(t)\ dt \le 0.9$. Since s is a pdf, we know that $\int_0^{\infty} s(t)\ dt = 1$, so the above graph is not a complete graph of $s(t)$ and there must be sessions longer than 75 minutes.