10. [11 points] The following parts are unrelated. Throughout this problem, justification is not required.

a. [3 points] Which of the following represent power series in the variable x? Circle **all** options which apply.

i. $x + x^3 + x^7 + x^{10} + \cdots$

ii. $\frac{1}{x-1} + \frac{1}{(x-1)^2} + \frac{1}{(x-1)^3} + \cdots$

iii. $1 + (x-1)^2 + (x-2)^3 + (x-3)^4 + \cdots$

iv. $x^{10} + 3x^3 + 12$

v. $\sum_{n=0}^{\infty} (\sin x)^n$

vi. $\sum_{n=0}^{\infty} 2^n x^n$

vii. $\sum_{n=1}^{\infty} \frac{x^{n/2}}{n}$

viii. None of these

b. [2 points] Suppose $p(x)$ is a probability density function (pdf) for a statistic which has mean value 4. Find the exact value of $\int_{-\infty}^{\infty} (2x+3)p(x)\, dx$.

Answer: 11

c. [6 points] For the following questions, determine if the statement is ALWAYS true, SOMETIMES true, or NEVER true, and circle the corresponding answer.

(i) Let $g(x)$ be a positive, decreasing, and continuous function. Suppose that for all n, $s_n = \int_n^{n+1} g(x)\, dx$. Then the sequence s_n converges.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

(ii) Let $b_n \geq 0$ and $c_n \geq 0$ for all n. Suppose that the series $\sum_{n=1}^{\infty} b_n$ converges and that the sequence c_n diverges. Then the series $\sum_{n=1}^{\infty} b_n c_n$ diverges.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

(iii) Let $F(x)$ be a cumulative distribution function (cdf) that is continuous for all x. Then the series $\sum_{n=1}^{\infty} (F(n+1) - F(n))$ converges.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**