2. [9 points] Let t (in minutes) denote the time Audrey waits for the *Bursley-Baits* shuttle to arrive. Observations show that the **probability density function** (pdf) of her wait time (in minutes) is of the form

$$p(t) = \begin{cases} 0, & t < 0, \\ 2\lambda t e^{-\lambda t^2} & t \geq 0, \end{cases}$$

where λ is a positive constant.
Throughout this problem, show all your work, and write your answers in exact form.

a. [5 points] Suppose that Audrey's median wait time for the Bursley-Baits shuttle is 1 minute. Find the value of λ.

Solution: If we call the cumulative distribution function (cdf) $P(x)$, then

$$\begin{aligned} P(x) &= \int_0^x p(t)\, dt \\ &= \int_0^x 2\lambda t e^{-\lambda t^2}\, dt \\ &= \int_0^{\lambda x^2} e^{-u}\, du \\ &= 1 - e^{-\lambda x^2}. \end{aligned}$$

Therefore, solving $P(1) = 0.5$ we get $\lambda = \ln 2$.

Answer: $\ln 2$

Sometimes Audrey takes the *Northwood Express* shuttle. For the Northwood Express shuttle, Audrey's wait time (in minutes) follows a **cumulative distribution function** (cdf) of the form

$$Q(t) = \begin{cases} 0, & t < 0, \\ 1 - (\lambda t + 1)e^{-\lambda t} & t \geq 0, \end{cases}$$

where λ is the **same** as in part **a**.

b. [2 points] When Audrey takes the Northwood Express shuttle, what is the fraction of rides where Audrey waits for 1 minute or less for the shuttle to arrive? Your final answer should not involve λ.

Solution: We have

$$Q(1) = 1 - (\lambda + 1)e^{-\lambda} = \frac{1}{2} - \frac{1}{2}\ln 2.$$

Answer: $\frac{1}{2} - \frac{1}{2}\ln 2$

c. [2 points] Audrey wants to choose the shuttle that has a lower median wait time. Which one should she choose? Explain your answer.

Circle one: [THE BURSLEY-BAITS SHUTTLE] THE NORTHWOOD EXPRESS SHUTTLE

Explanation:

Solution: Since $Q(1) < 0.5$, we know that the median of the wait time for Northwood shuttle is greater than 1, while the median of the wait time for the Bursley-Baits shuttle is equal to 1. Therefore, Audrey should choose the Bursley-Baits shuttle.