4. [7 points] **Compute** the value of the following improper integral if it converges. If it does not converge, use a **direct computation** of the integral to show its divergence. Be sure to show your full computation, and be sure to use **proper notation**.

$$\int_0^{\sqrt{8}} \frac{x}{(8-x^2)^{1/3}}\,\mathrm{d}x$$

Solution:
We first rewrite this improper integral as a limit.

$$\int_0^{\sqrt{8}} \frac{x}{(8-x^2)^{1/3}}\,\mathrm{d}x = \lim_{b\to\sqrt{8}^-} \int_0^{b} \frac{x}{(8-x^2)^{1/3}}\,dx$$

We then use the substitution $u = 8 - x^2$ to see,

$$\begin{aligned}\lim_{b\to\sqrt{8}^-} \int_0^{b} \frac{x}{(8-x^2)^{1/3}}\,dx &= \lim_{b\to\sqrt{8}^-} -\frac{1}{2}\int_8^{8-b^2} \frac{du}{u^{1/3}}\,dx \\ &= \lim_{b\to\sqrt{8}^-} -\frac{3}{4}u^{2/3}\Big|_8^{8-b^2} \\ &= \lim_{b\to\sqrt{8}^-} \frac{3}{4}\left(8^{2/3} - (8-b^2)^{2/3}\right) \\ &= \frac{3}{4}(4-0) = 3.\end{aligned}$$

Therefore the integral converges to 3.

Circle one: **Diverges** | **Converges to** 3