5. [7 points] Determine whether the following improper integral converges or diverges and circle the corresponding word. **Fully justify** your answer including using **proper notation** and showing mechanics of any tests you use. You do not need to calculate the value of the integral if it converges.

$$\int_2^\infty \frac{x+1}{5x^3+3x^3\sin x}\,\mathrm{d}x$$

Circle one: **Converges** **Diverges**

Justification:

Solution: We use Direct Comparison Test (DCT) to show the convergence of the integral. Note that for $x \geq 2$,

$$\frac{x+1}{5x^3+3x^3\sin x} \leq \frac{2x}{2x^3} = \frac{1}{x^2}$$

and that by p-test (with $p = 2$),

$$\int_2^\infty \frac{1}{x^2}\,dx$$

converges. Therefore, by Direct Comparison Test,

$$\int_2^\infty \frac{x+1}{5x^3+3x^3\sin x}\,\mathrm{d}x$$

also converges.