7. [15 points]

a. [7 points] Determine if the following series converges or diverges using the **Limit Comparison Test**, and circle the corresponding word. **Fully justify** your answer including using **proper notation** and showing mechanics of any tests you use.

$$\sum_{n=1}^{\infty} \frac{3n-2}{\sqrt{4n^3-5n^2+3}}$$

Circle one: **Converges** **Diverges**

*Justification (using the **Limit Comparison Test**):*

Solution: We aim to compare with the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$. We see that

$$\lim_{n\to\infty} \frac{\frac{3n-2}{\sqrt{4n^3-5n^2+3}}}{\frac{1}{n^{1/2}}} = \lim_{n\to\infty} \frac{3n^{3/2}-2n^{1/2}}{\sqrt{4n^3-5n^2+3}} = \lim_{n\to\infty} \frac{3n^{3/2}}{2n^{3/2}} = \frac{3}{2},$$

and so the Limit Comparison Test does apply.
By the p-test for series (with $p = \frac{1}{2}$),

$$\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$$

diverges. Therefore, by the Limit Comparison Test,

$$\sum_{n=1}^{\infty} \frac{3n-2}{\sqrt{4n^3-5n^2+3}}$$

also diverges.

b. [8 points] Determine if the following series absolutely converges, conditionally converges, or diverges, and circle the corresponding word. **Fully justify** your answer including using **proper notation** and showing mechanics of any tests you use.

$$\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}+1}$$

Circle one: **Converges Absolutely** **Converges Conditionally** **Diverges**

Justification:

Solution: First of all, note that

$$\frac{1}{\sqrt{n}+1} \geq \frac{1}{2\sqrt{n}}$$

for $n \geq 1$. Since

$$\sum_{n=1}^{\infty}\frac{1}{2\sqrt{n}}$$

diverges by the p-test for series (with $p = \frac{1}{2}$), we see by the Comparison Test that

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+1}$$

diverges.

On the other hand, we note that

$$\frac{1}{\sqrt{n}+1}$$

is monotone decreasing and approaches 0 as $n \to \infty$, and so by the Alternating Series Test (AST),

$$\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}+1}$$

converges.

As a result, $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}+1}$ **converges conditionally**.