

1. (2 pts each) Circle true or false. No explanation necessary.

**True** or **False**: The Taylor series centered at  $x = 2$  of  $e^x$  is

$$1 + (x - 2) + \frac{(x - 2)^2}{2!} + \frac{(x - 2)^3}{3!} + \frac{(x - 2)^4}{4!} + \frac{(x - 2)^5}{5!} + \dots$$

This is  $e^{x-2}$ . If you multiplied all the terms by  $e^2$ , you'd have  $e^x$  centered at 2.

**True** or **False**: If  $\lim_{n \rightarrow \infty} a_n = 0$ , then  $\sum_{n=0}^{\infty} a_n$  converges.

The harmonic series ( $a_n = 1/n$ ) is a counterexample.

**True** or **False**: The fourth derivative of

$$f(x) = 6 + 6x + 6x^2 + 6x^3 + \frac{x^4}{4} + 6x^5 + 6x^6 + 6x^7 + \dots$$

at the point  $x = 0$  is 6.

The only term that matters is the  $x^4$ . After 4 derivatives it will be  $(\frac{1}{4})(4)(3)(2)(1) = 6$ .

2. (8 pts) Art inspires art: Whenever artists create art, other people have the opportunity to take inspiration and create more art. These responses can inspire still other works, and so on.

Assume that creating any number  $x$  of artistic works will inspire  $.8x$  more artistic works (fractional works are ok—don't round). If you produce 75 sculptures, for how many total works can you claim indirect influence? In other words, how many artistic works are there, in total, which are part of your collection, inspired by your collection, inspired by those inspired by your collection, etc? Show your work carefully. Do not round, and do not estimate.

The works you can take credit for are:

Works you create	75
Works inspired by works you create	$75(.8)$
Works inspired by the works on the line above	$75(.8)(.8)$
Works inspired by the works on the line above	$75(.8)(.8)(.8)$
Etc.	

So the total is

$$75(1 + .8 + .8^2 + .8^3 + \dots) = \frac{75}{1 - .8} = \boxed{375 \text{ works of art}}.$$