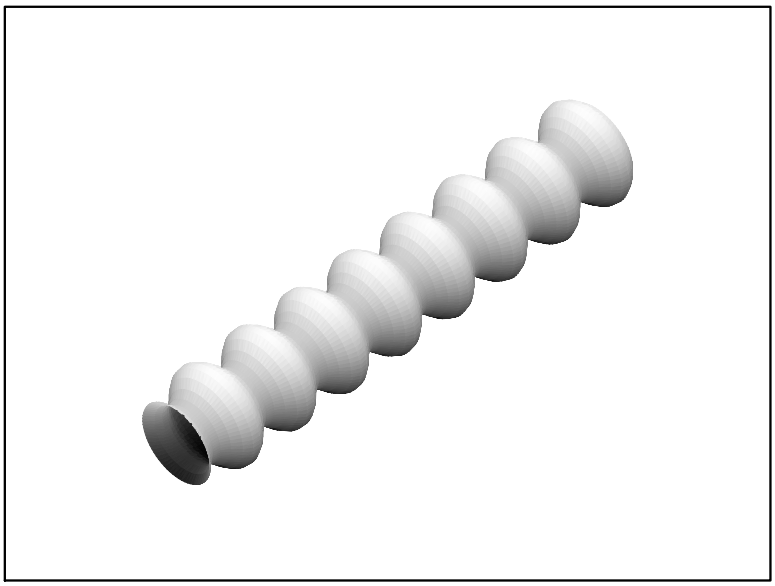
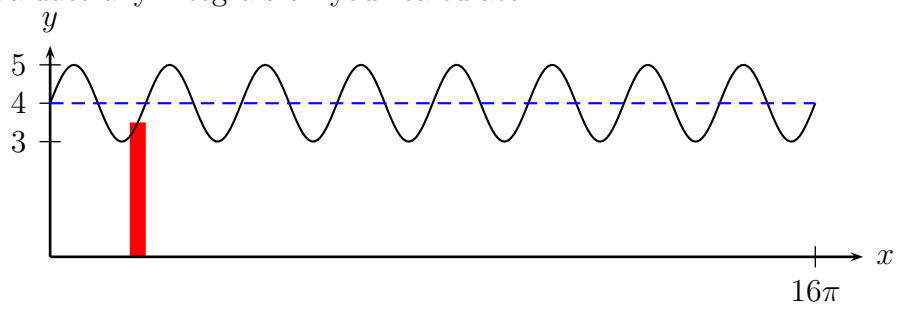
3. (12 pts) A decorative table leg (see diagram) is manufactured so that it is the volume of rotation of the function $f(x) = 4 + \sin(x)$ between $x = 0$ and $x = 16\pi$.



a. (8 pts) What is the volume of the table leg? Show all work, but (on this problem only) you may evaluate any integrals on your calculator.



The red rectangle, when rotated around the x-axis, represents a slice of the chair leg. We have

$$\text{Volume of slice} = \pi(4 + \sin(x))^2 \Delta x$$

so

$$\text{Total volume} = \int_0^{16\pi} \pi(4 + \sin(x))^2 \, dx \approx 2606 \, \text{in}^2.$$

If you do the integral algebraically you'll find that the exact volume is $264\pi^2$.

b. (4 pts) Here's a plausible shortcut: Replace the complicated shape with a cylinder with height 16π and radius 4 (because the average radius above is 4), and apply the volume formula for a cylinder. Is the shortcut valid? Explain briefly.

No, not valid. In this case, the shortcut would give an answer of

$$\pi r^2 h = \pi(4)^2(16\pi) = 256\pi^2 \approx 2527,$$

which we know to be too low. The peaks and valleys in the silhouette above correspond to bumps and grooves in the chair leg. The problem is that while the peaks and valleys are symmetric (i.e., they have the same area, so eliminating them both leaves the area of the sihouette the same), the bumps and grooves are not. The bumps have more volume than the grooves, because their slices have larger radii.