5. (18 pts) Fred likes to juggle. So does jason. The number of minutes Fred can juggle five balls without dropping one is a random variable, with probability density function $f(t) = 0.8e^{-0.8t}$. Similarly, the function $j(t) = 1.5e^{-1.5t}$ describes jason's skill. Here t is time *in minutes*.

a. (2 pts) Find $\int_0^\infty f(t)dt$. No need to show work.

That integral must be $\boxed{1}$, because f is a pdf.

b. (5 pts) What percentage of jason's juggling attempts are "embarrassing," meaning they last for 10 seconds or less? Show your work.

10 seconds is $\frac{1}{6}$ minute. So the proportion we want is

$$\int_0^{1/6} j(t)\,dt = \int_0^{1/6} 1.5e^{-1.5t}\,dt = -e^{-1.5t}\Big|_0^{1/6} = -e^{-1.5(1/6)} - (-e^0) = 1 - e^{-1/4} \approx \boxed{22\%}.$$

c. (6 pts) How long can Fred juggle, on average? Show your work.

$$\text{Fred's Average Time} = \int_0^\infty tf(t)\,dt = \int_0^\infty 0.8te^{-0.8t}\,dt.$$

Parts:

$$\begin{array}{ll} u = t & v = -e^{-0.8t} \\ u' = 1 & v' = 0.8e^{-0.8t} \end{array}$$

which yields:

$$\begin{aligned} \int 0.8te^{-0.8t}\,dt &= \int uv'\,dt = uv - \int u'v\,dt \\ &= -te^{-0.8t} - \int -e^{-0.8t}\,dt = -te^{-0.8t} - \frac{1}{0.8}e^{-0.8t} + C \\ &= -(t+1.25)e^{-0.8t} + C. \end{aligned}$$

Therefore

$$\begin{aligned} \text{Fred's Average Time} &= \lim_{b\to\infty} -(t+1.25)e^{-0.8t}\Big|_0^b \\ &= \left(\lim_{b\to\infty}\left(-(b+1.25)e^{-0.8b}\right)\right) - \left(-(0+1.25)e^{-0.8(0)}\right) \\ &= 1.25 - \lim_{b\to\infty}(b+1.25)e^{-0.8b}. \end{aligned}$$

Exponentials always beat polynomials, so $e^{-0.8b}$ goes to 0 faster than $b+1.25$ goes to infinity. So the last limit is 0, which makes Fred's average time $\boxed{1.25 \text{ minutes}}$.

d. (5 pts) Who is the better juggler? Give a good reason for your decision.

Fred's average time was $1.25 = 1/0.8$, so Jason's will be $.67 = 1/1.5$. So $\boxed{\text{Fred}}$ has a better average.