**7.** (16 pts)

**a.** (8 pts) Find the Taylor series expansion for the function $\ln(2+x)$ centered at the point $x = 0$.

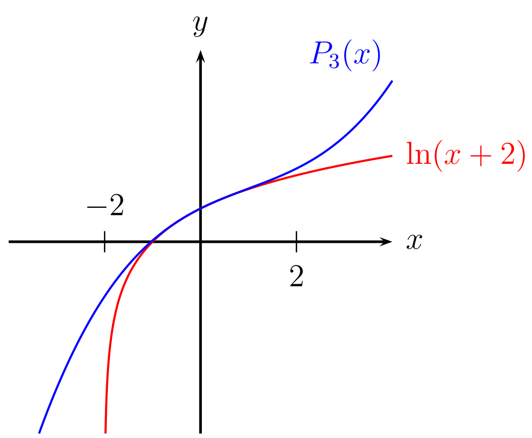
| $n$ | $f^{(n)}(x)$ | $f^{(n)}(0)$ |
|---|---|---|
| 0 | $\ln(2+x)$ | $\ln 2$ |
| 1 | $(2+x)^{-1}$ | $2^{-1}$ |
| 2 | $(-1)(2+x)^{-2}$ | $-1! \cdot 2^{-2}$ |
| 3 | $(-2)(-1)(2+x)^{-3}$ | $2! \cdot 2^{-3}$ |
| 4 | $(-3)(-2)(-1)(2+x)^{-4}$ | $-3! \cdot 2^{-4}$ |
| $n$ | $-(n-1)\cdots(-1)(2+x)^{-n}$ | $(-1)^{n-1}(n-1)! \cdot 2^{-n}$ |

So the Taylor series is

$$\ln(2+x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n = \ln 2 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(n-1)! \cdot 2^{-n}}{n!}x^n = \ln 2 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot 2^{-n}x^n}{n}$$

$$= \boxed{\ln 2 + \frac{x}{2} - \frac{1}{2}\left(\frac{x}{2}\right)^2 + \frac{1}{3}\left(\frac{x}{2}\right)^3 - \frac{1}{4}\left(\frac{x}{2}\right)^4 + \cdots}$$

**b.** (4 pts) Using your calculator, graphically approximate the domain of convergence of this Taylor series. Accurately sketch a graph which suggests how you got your answer.




Graphing $\ln(x+2)$ together with the third-order Taylor polynomial

$$P_3(x) = \ln(2) + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{24}$$

yields the picture on the left. It suggests that the domain of convergence is $(-2, 2)$, since that is the region on which the polynomial approximates the function well. Adding more terms to the polynomial gives a graph which fits more closely on $(-2, 2)$ and diverges more quickly elsewhere. So this is probably the domain of convergence.

**c.** (4 pts) It is claimed "One way to approximate $\ln(10)$ is to plug in 8 to the series above, using the first dozen, hundred, or even more terms. The more terms you take, the better your approximation of $\ln(10)$." Explain why this plan will (or will not) work.

That would work if 8 were in the domain of convergence of the Taylor series. But plugging 8 into the Taylor series will give a silly nonconvergent mess, and so that would not help calculate $\ln(10)$. We could, however, calculate $\ln(\pi)$ by plugging 1.14159 into the formula.