

8. (10 pts) On this problem you must show your work and use exact methods. That is, calculator approximations are insufficient.

Find two values of x for which

$$x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \frac{x^{18}}{9!} - \frac{x^{22}}{11!} + \dots = 1.$$

Despite the high number of points and the ugly formula, there's really only one idea: The formula above is the Taylor series for $\sin(x)$, except that the powers of x are twice as high as they should be. That means that the ugly expression above is in fact $\sin(x^2)$. Simplifying: $\sin(x^2) = 1$, so

$$x^2 \in \left\{ \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \dots \right\}$$

which means

$$x \in \left\{ \pm\sqrt{\frac{\pi}{2}}, \pm\sqrt{\frac{5\pi}{2}}, \pm\sqrt{\frac{9\pi}{2}}, \dots \right\}.$$