1. (4 points) Find the sum of the infinite series

\[ 2 + \left( \frac{2}{3} \right)^2 + \left( \frac{2}{3} \right)^3 + \cdots + \left( \frac{2}{3} \right)^n + \cdots \]

\[ 2 + \left( \frac{2}{3} \right)^2 + \left( \frac{2}{3} \right)^3 + \cdots + \left( \frac{2}{3} \right)^n + \cdots = 2 + \left( \frac{2}{3} \right)^2 \left[ 1 + \left( \frac{2}{3} \right)^2 + \left( \frac{2}{3} \right)^3 + \cdots + \left( \frac{2}{3} \right)^n + \cdots \right] \]

\[ = 2 + \left( \frac{2}{3} \right)^2 \left[ \frac{1}{1 - \frac{2}{3}} \right] = 2 + \frac{4}{3} = \frac{10}{3}. \]

2. (4 points) Does the infinite series \( \sum_{n=1}^{\infty} \frac{n^3}{n^5 + 1} \) converge? Explain why or why not.

The infinite series \( \sum_{n=1}^{\infty} \frac{1}{n^2} \) converges and the given infinite series has smaller positive terms because \( 0 < \frac{n^3}{n^5 + 1} \leq \frac{n^3}{n^5} = \frac{1}{n^2} \). Therefore, the series converges by the comparison test.

3. (8 points) If the fourth degree Taylor polynomial approximating a function \( f \) near \( x = -1 \) is \( P_4(x) = 2 - 3(x + 1) - (x + 1)^3 + 4(x + 1)^4 \), then

(a) The linear approximation to \( f \) near \( x = -1 \) is \( 2 - 3(x + 1) \).

(b) \( f'''(−1) = -6 \).