1. (4 points) Find the sum of the infinite series

$$2+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^3+\cdots+\left(\frac{2}{3}\right)^n+\ldots.$$

$$2+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^3+\cdots+\left(\frac{2}{3}\right)^n+\ldots=2+\left(\frac{2}{3}\right)^2\left[1+\left(\frac{2}{3}\right)+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^3+\cdots+\left(\frac{2}{3}\right)^n+\ldots\right]$$

$$=2+\left(\frac{2}{3}\right)^2\left[\frac{1}{1-\frac{2}{3}}\right]=2+\frac{4}{3}=\frac{10}{3}.$$

2. (4 points) Does the infinite series $\sum_{n=1}^{\infty}\frac{n^3}{n^5+1}$ converge? Explain why or why not.

The infinite series $\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges and the given infinite series has smaller positive terms because $0<\frac{n^3}{n^5+1}\leq\frac{n^3}{n^5}=\frac{1}{n^2}$. Therefore, the series converges by the comparison test.

3. (8 points) If the fourth degree Taylor polynomial approximating a function f near $x=-1$ is $P_4(x)=2-3(x+1)-(x+1)^3+4(x+1)^4$, then

(a) The linear approximation to f near $x=-1$ is $\boxed{2-3(x+1)}$.

(b) $f'''(-1)=$ $\boxed{-6}$.