10. (10 points) The *electric potential* is a quantity of great importance in electrostatics. The electric potential $V(R)$ at a distance R along the axis perendicular to the center of a charged disk with radius 1 is given by

$$V(R) = C\left(\sqrt{R^2+1} - R\right)$$

where C is a constant that depends on the choice of units that are being used.

(a) Show that for large numbers R,

$$V(R) \approx \frac{C}{2R}.$$

(Hint: $\sqrt{R^2+1} = R\sqrt{1+\frac{1}{R^2}}$ and remember that R is large.)

As the hint says, $\sqrt{R^2+1} = R\sqrt{1+\frac{1}{R^2}} = R(1+\frac{1}{R^2})^{\frac{1}{2}}$**.**

Recall (or calculate) that for $-1 < x < 1$**, we have** $(1+x)^{\frac{1}{2}} = 1+\frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \ldots$

Thus for R **large (enough so that** $-1 < \frac{1}{R^2} < 1$**) we have**

$(1+\frac{1}{R^2})^{\frac{1}{2}} = 1 + \frac{1}{2}\frac{1}{R^2} - \frac{1}{8}(\frac{1}{R^2})^2 + \frac{1}{16}(\frac{1}{R^2})^3 - \ldots = 1 + \frac{1}{2}\frac{1}{R^2} - \frac{1}{8}\frac{1}{R^4} + \frac{1}{16}\frac{1}{R^6} - \ldots$

So $R(1+\frac{1}{R^2})^{\frac{1}{2}} = R + \frac{1}{2}\frac{1}{R} - \frac{1}{8}\frac{1}{R^3} + \frac{1}{16}\frac{1}{R^5} - \ldots$

Therefore $R(1+\frac{1}{R^2})^{\frac{1}{2}} - R = \frac{1}{2}\frac{1}{R} - \frac{1}{8}\frac{1}{R^3} + \frac{1}{16}\frac{1}{R^5} - \ldots$

So $C(R(1+\frac{1}{R^2})^{\frac{1}{2}} - R) = \frac{C}{2}\frac{1}{R} - \frac{C}{8}\frac{1}{R^3} + \frac{C}{16}\frac{1}{R^5} - \ldots$

And so we have that for large numbers R**, we can approximate** $C(R(1+\frac{1}{R^2})^{\frac{1}{2}} - R)$ **by** $\frac{C}{2}\frac{1}{R}$**.**

(b) Approximately how large should R be in order that the error in the approximation of $V(R)$ by $C/2R$ is less than 1% of $V(R)$?

For large R**, the error in the approximation of** $V(R)$ **by** $C/2R$ **is approximately** $\frac{-C}{8R^3}$**.**

So, we want to apporoximately solve $V(R)\frac{1}{100} = \frac{C}{8R^3}$ **for** R**.**

This is approximately the same as approximately solving $\frac{C}{2R}\frac{1}{100} = \frac{C}{8R^3}$ **for** R**.**

So we want to solve $4R^2 = 100$ **and thus**

R should approximately be greater than or equal to 5