1. (4 points) Find the sum of the infinite series

$$2 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots + \left(\frac{2}{3}\right)^n + \ldots.$$

$$2 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots + \left(\frac{2}{3}\right)^n + \ldots = 2 + \left(\frac{2}{3}\right)^2 \left[1 + \left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots + \left(\frac{2}{3}\right)^n + \ldots\right]$$

$$= 2 + \left(\frac{2}{3}\right)^2 \left[\frac{1}{1 - \frac{2}{3}}\right] = 2 + \frac{4}{3} = \frac{10}{3}.$$

2. (4 points) Does the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{n^5 + 1}$ converge? Explain why or why not.

The infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges and the given infinite series has smaller positive terms because $0 < \dfrac{n^3}{n^5 + 1} \leq \dfrac{n^3}{n^5} = \dfrac{1}{n^2}$. Therefore, the series converges by the comparison test.

3. (8 points) If the fourth degree Taylor polynomial approximating a function f near $x = -1$ is $P_4(x) = 2 - 3(x+1) - (x+1)^3 + 4(x+1)^4$, then

(a) The linear approximation to f near $x = -1$ is $\boxed{2 - 3(x+1)}$.

(b) $f'''(-1) = \boxed{-6}$.