1. (4 points) Find the sum of the infinite series

\[
2 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots + \left(\frac{2}{3}\right)^n + \cdots
\]

\[
2 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots + \left(\frac{2}{3}\right)^n + \cdots = 2 + \left(\frac{2}{3}\right)^2 \left[1 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots + \left(\frac{2}{3}\right)^n + \cdots \right]
\]

\[
= 2 + \left(\frac{2}{3}\right)^2 \left[\frac{1}{1 - \frac{2}{3}}\right] = 2 + \frac{4}{3} = \frac{10}{3}.
\]

2. (4 points) Does the infinite series \(\sum_{n=1}^{\infty} \dfrac{n^3}{n^5 + 1}\) converge? Explain why or why not.

The infinite series \(\sum_{n=1}^{\infty} \dfrac{1}{n^2}\) converges and the given infinite series has smaller positive terms because 0 < \(\dfrac{n^3}{n^5 + 1} \leq \dfrac{n^3}{n^5} = \dfrac{1}{n^2}\). Therefore, the series converges by the comparison test.

3. (8 points) If the fourth degree Taylor polynomial approximating a function \(f\) near \(x = -1\) is \(P_4(x) = 2 - 3(x + 1) - (x + 1)^3 + 4(x + 1)^4\), then

(a) The linear approximation to \(f\) near \(x = -1\) is \(\boxed{2 - 3(x + 1)}\).

(b) \(f'''(-1) = \boxed{-6}\).