6. (12 points) A team of biologists is interested in the ability of certain birds to migrate great distances with little rest. The biologists are monitoring a flock of birds known to migrate after spending the winter in the warm climes of the Okefenokee swamp. The location of the flock, in $x(t)$ hundreds of miles north, and $y(t)$ hundreds of miles east of the base camp of the biologists, t days after their departure from the Okefenokee swamp is given by

$$x(t) = 3t + \frac{1}{2},$$
$$y(t) = t^{\frac{3}{2}} + \frac{1}{5}.$$

(a) Where is the Okefenokee swamp in relation to the base camp of the biologists?

> At $t = 0$, the birds are at the swamp, so it is located $x(0) = .5$ hundred miles north and $y(0) = .2$ hundred miles east of the base camp of the biologists. That is, 50 miles north and 20 miles east.

(b) Is there ever a time when the flock of birds is travelling due North-East? If so, when? If not, explain why not.

> The flock is travelling north-east at time t if $x'(t) = y'(t) > 0$. Since $x'(t) = 3$ and $y'(t) = \frac{3}{2}t^{1/2}$, this occurs when $3 = \frac{3}{2}t^{1/2}$ or when $t = 4$ days.

(c) Is the flock of birds constantly moving throughout the first three days of their journey? Why or why not?

> The velocity of the flock at time t is $v(t) = \sqrt{(x'(t))^2 + (y'(t))^2} = \sqrt{9 + \frac{9}{4}t} \geq 3 > 0$, so the flock is constantly in motion during all the days it is travelling.

(d) How far does the flock of birds travel in the first three days of their journey?

> Since the rate of change of distance travelled with respect to time is the velocity, the distance travelled during the first three days is given by the integral.
>
> $$\int_0^3 v(t)\, dt = \int_0^3 \sqrt{9 + \frac{9}{4}t}\, dt$$
>
> One can calculate this integral, either by using the fact that an antiderivative for the integrand is $8(1 + \frac{t}{4})^{3/2}$ and applying the fundamental theorem of calculus, or by using numerical integration. The distance travelled is $(\sqrt{7})^3 - 8 \approx 10.52$ hundred miles, or about 1052 miles.