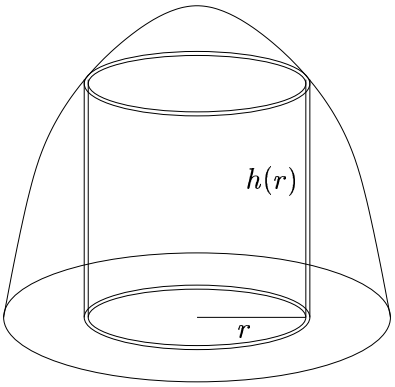
9. (10 points) Paleontologists have found some interesting fossils embedded in a stalagmite in a cave. To help determine the age of the fossils they want to measure the mass of the stalagmite.

At its base, the stalagmite has a radius of 50 cm and its height $h(r)$, r cm from its center is $\dfrac{r^2 - 100r + 2500}{50}$ cm. The density of the limestone from which the stalagmite is made r cm from its center is $\dfrac{50 + r}{50}$ g/cm^3. (See the figure.)

To help determine the age of the fossils, the paleontologists want to calculate the mass of the stalagmite. What is this mass?



Pictured thin clinder has walls of thickness dr.
Volume of pictured thin cylinder $= 2\pi r h(r)\, dr$.
Constant density on this cylinder $= \dfrac{50 + r}{50}$.
Mass of this cylindrical shell $= \dfrac{50 + r}{50} 2\pi r h(r)\, dr$.
Total mass of stalagmite $= \displaystyle\int_0^{50} \frac{50 + r}{50} 2\pi r h(r)\, dr = \int_0^{50} \frac{50 + r}{50} 2\pi r \frac{r^2 - 100r + 2500}{50}\, dr$.
Note that integrand $= \dfrac{2\pi}{2500}(r^4 - 50r^3 - 50^2 r^2 + 50^3 r)$.
Calculating the integral, either by computing an antiderivative and using the fundamental theorem of calculus, or but using a calculator, one finds that:

The mass of the stalagmite is $\dfrac{7 \cdot 50^3 \cdot \pi}{30} == \dfrac{87,500\pi}{3} = 91629.786$ **g.**