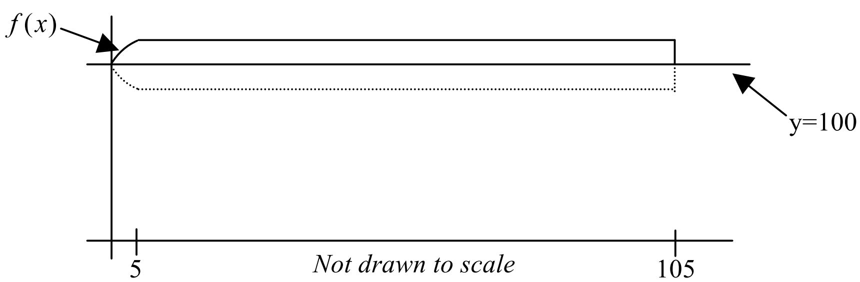
2. (60 points) Several different forces act on a submerged submarine and cause it to rise and/or fall. In this problem, we will use a simplified model of a submarine to explore some of these forces. To construct our model, we revolve the graph of

$$f(x)=\begin{cases} 100+\sqrt{5}e^{0.5}\sqrt{x}e^{-0.1x} & 0\le x\le 5 \\ 105 & 5< x\le 105 \end{cases}$$

around $y=100$ (see picture below). Note that all units in the horizontal and vertical directions are measured in meters. We will also assume that ocean water has density of $1025\,{}^{kg}\!/\!_{m^3}$ and that the density of material inside the submarine is a constant represented by the symbol δ. Additionally, the acceleration due to gravity is $9.8\,{}^{m}\!/\!_{sec^2}$.



a. The *force due to buoyancy* is an upward force equal to the weight of the water displaced by the volume of the submarine. Find the volume of water displaced by the sub (i.e. the volume of the submarine) and the resulting force due to the buoyancy.

b. Find the center of mass of the nose of the submarine (i.e. the shape of the first 5 meters of our model). Note *it is only necessary to set up, but not calculate*, (an) integral(s).

3. (60 points) The following two questions refer to the submarine described in problem #2.
   a. The buoyancy properties of the empty submarine described in problem 1 cause the submarine to begin moving upward through the ocean water. This motion, in conjunction with the ocean water, creates a *damping force* that begins to slow the submarine. Assume that the damping force is proportional to the square of the velocity of the submarine, and that when the velocity is 5m/s the force is 100N. For our model submarine, the velocity at t seconds can be described by $v(t) = \left(25 - 25\sin\left(\frac{\pi t}{60}\right)\right)^{\frac{1}{3}}$ (in meters per second). Find the amount of work the damping force does on the submarine over the first 30 seconds of motion.