4. (50 points) The *Erlang k-distribution* is a probability distribution often used in mathematical modeling when events happen at a roughly (but not exactly) constant rate. It is a good model for the wait times at a telephone switchboard when calls come in on average every λ seconds. In this case, the wait time for the next k telephone calls has a probability density function that is the Erlang k-distribution

$$f_{k,\lambda}(x) = \begin{cases} \dfrac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!} & x \geq 0 \\ 0 & x < 0 \end{cases}.$$

a. For $x \geq 0$, the Erlang k- distribution has the non-obvious cumulative distribution function,

$$C_{k,\lambda}(x) = 1 - \sum_{n=0}^{k-1} \frac{e^{-\lambda x}(\lambda x)^n}{n!}.$$

Using an appropriate test, show that the sum in the cumulative distribution function converges as $k \to \infty$, thus verifying that $C_{k,\lambda}(x)$ is finite. You may assume $x = 1$ and $\lambda = 3$. Note: saying "the distribution function must be finite, therefore it converges" will not be given credit.

b. A call arrives at the switchboard at 2:38:06pm. Assuming $k=1$ and $\lambda=3$ seconds, find the probability that the next phone call comes in between 2:38:08pm and 2:38:09pm.