2. (b) Find the center of mass of the nose of the submarine (i.e. the shape of the first 5 meters of our model). Note *it is only necessary to set up, but not calculate*, (an) integral(s).

$$\bar{x} = \frac{\int_0^5 x\delta A_x(x)\,dx}{\text{mass}} = \frac{\int_0^5 x\delta\pi\left(\sqrt{5}e^{0.5}\sqrt{x}e^{-0.1x}\right)^2\,dx}{\text{mass}} = \frac{5e\pi\delta\int_0^5 x^2e^{-0.2x}\,dx}{\text{mass}}$$

where

$$\text{mass} = 1025(2250\pi + 125\pi e) = 8339451.5\,\text{kg}.$$

By symmetry, $\bar{y} = \bar{z} = 0$. In this case, I'm assuming the z direction is perpendicular to the usual x and y axes.

3. (60 points) The following questions refer to the submarine described in problem #2.

(a) The buoyancy properties of the empty submarine described in problem 2 cause the submarine to begin moving upward through the ocean water. This motion, in conjunction with the ocean water, creates a damping force that begins to slow the submarine. Assume that the damping force is proportional to the square of the velocity of the submarine, and that when the velocity is 5 m/s the force is 100 N. For our model submarine, the velocity at t seconds can be described by

$$v(t) = \left(25 - 25\sin\left(\frac{\pi t}{60}\right)\right)^{\frac{1}{3}} \quad \text{meters per second.}$$

Find the amount of work the damping force does on the submarine over the first 30 seconds of motion.

From the problem statement, the damping force is kv^2 where k is the proportionality constant. Since the force is 100 N when the velocity is 5 m/s, we solve and find that

$$k = \frac{100\,\text{N}}{(5\,\text{m/s})^2} = 4\,\text{kg/m}.$$

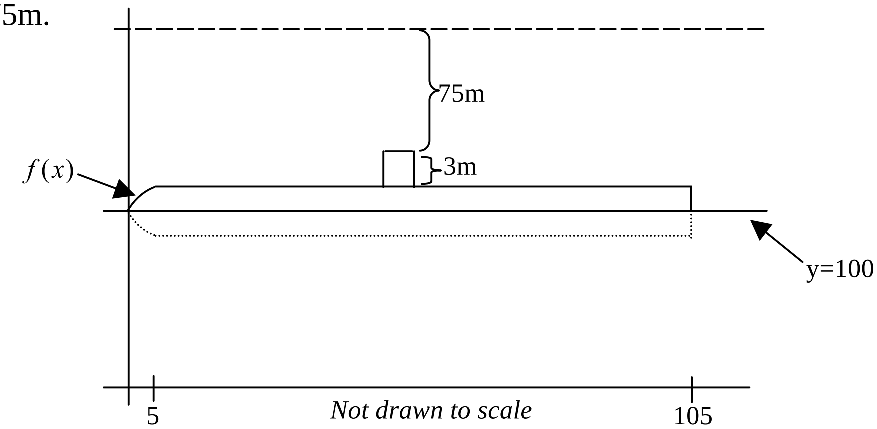
The distance travelled from time t to time $t + \Delta t$ is approximately $v(t)\Delta t$. Thus the work done over that slice of time is

$$\text{Force}\cdot\text{Distance} = (kv^2)(v\Delta t) = kv^3\Delta t = 4\left(25 - 25\sin\left(\frac{\pi t}{60}\right)\right)\Delta t$$

which means the total work is

$$\begin{aligned}\int_0^{30} 4\left(25 - 25\sin\left(\frac{\pi t}{60}\right)\right)dt &= 100\int_0^{30}\left(1 - \sin\left(\frac{\pi t}{60}\right)\right)dt = 100\left[t + \frac{60}{\pi}\cos\left(\frac{\pi t}{60}\right)\right]_0^{30} \\ &= 100\left[\left(30 + \frac{60}{\pi}\cos\frac{\pi}{2}\right) - \left(0 + \frac{60}{\pi}\cos 0\right)\right] \\ &= 100(30 - 60/\pi) \approx \boxed{1090.14\,\text{Joules}}.\end{aligned}$$

b. The *sail* of a submarine is a tower that houses the command and communications center, periscope(s), radar and antennae. We will additionally assume our model submarine has a sail that is a circular cylinder with radius of 2m and a height of 3m. Determine the total force on the sail (i.e. top and side) due to water pressure when the top of the sail is at a depth of 75m.



Pressure= mass density x acceleration due to gravity x depth.
Force=pressure x area.
So Force= mass density x acceleration due to gravity x depth x area.

Force on the side of the sail.
We will slice the sail vertically, with h = 0 located at the bottom of the sail. We also assume that up is in the positive direction.
This yields circular slices. So $Force_{slice} = 1025 \cdot 9.8 \cdot (78 - h) \cdot (2\pi \cdot 2\Delta h)$, where $(2\pi \cdot 2\Delta h)$ is the area of a circular strip of radius 2 and height Δh. The total force is found by adding up the force on each slice and taking the limit as $\Delta h \to 0$. Symbolically, this is

$$\lim_{\Delta h \to 0} \sum 1025 \cdot 9.8 \cdot (78 - h) \cdot (2\pi \cdot 2\Delta h) = 1025 \cdot 9.8 \cdot 4\pi \int_0^3 (78 - h)\, dh$$

Finding anti-derivatives yields $1025 \cdot 9.8 \cdot 4\pi \left[78h - \frac{h^2}{2} \right]_0^3$ =40,180

$40{,}180\pi \cdot 229.5 = 9{,}221{,}310\pi = 28{,}969{,}599.752$ Units are Newtons.

Force on the top. Depth is 75m and area is $\pi(2)^2$. So force on top is
$1025 \cdot 9.8 \cdot 75 \cdot 4\pi = 3{,}013{,}500\pi = 9{,}467{,}189.462N$.

The total is $12{,}234{,}810\pi = 38{,}436{,}789.21N$