6. (48 points) Given \( \sum_{n=1}^{\infty} a_n = 0.72 \), \( b_n = n^2 \), \( c_n = (n+1)^3 \) determine whether or not the following statements are True or False. To receive full credit, you must justify your decision with a calculation, a sentence or two, or a relevant picture that illustrates your thinking.

a. \( \lim_{n \to \infty} a_n = 0.72 \). FALSE. Individual terms of a sequence must approach zero in order for the series to converge.

b. \( a_{n+1} < a_n \) for all \( n \). FALSE. The sequence \( a_1 = 0, a_2 = .72, a_3 = 0, a_4 = 0, a_5 = 0, \) etc. has sum \( \sum_{n=1}^{\infty} a_n = 0.72 \). But \( a_{n+1} < a_n \) does not hold for all \( n \).

c. \( \lim_{n \to \infty} s_n = 0.72 \) where \( s_n = a_1 + a_2 + \ldots + a_n \). TRUE. This is the definition of convergence of a series.

d. \( \lim_{n \to \infty} \frac{b_n}{c_n} \) converges. TRUE. \( \lim_{n \to \infty} \frac{b_n}{c_n} = \lim_{n \to \infty} \frac{n^2}{(n+1)^3} = 0 \) since the denominator has a polynomial with larger degree than the polynomial in the numerator.

e. \( \sum_{n=1}^{\infty} \frac{b_n}{c_n} \) converges. FALSE. \( \sum_{n=1}^{\infty} \frac{n^2}{(n+1)^3} > \sum_{n=1}^{\infty} \frac{1}{n} \). But \( \sum_{n=1}^{\infty} \frac{1}{n} \) diverges \( (p = 1) \) and by comparison test so must \( \sum_{n=1}^{\infty} \frac{b_n}{c_n} \).

f. \( \sum_{n=1}^{\infty} (-1)^n \frac{b_n}{c_n} \) converges. TRUE. This is an alternating series. \( \lim_{n \to \infty} \frac{b_n}{c_n} = \lim_{n \to \infty} \frac{n^2}{(n+1)^3} = 0 \) and \( \frac{n^2}{(n+1)^3} \) decreases to zero as \( n \) increases. So by alternating series test, the series converges.