6. (48 points) Given $\sum_{n=1}^{\infty} a_n = 0.72$, $b_n = n^2$, $c_n = (n+1)^3$ determine whether or not the following statements are is True or False. To receive full credit, you must justify your decision with a calculation, a sentence or two, or a relevant picture that illustrates your thinking.

   a. $\lim_{n\to\infty} a_n = 0.72$. FALSE. Individual terms of a sequence must approach zero in order for the series to converge.

   b. $a_{n+1} < a_n$ for all *n*. FALSE. The sequence $a_1 = 0$, $a_2 = .72$, $a_3 = 0$, $a_4 = 0$, $a_5 = 0$, etc. has sum $\sum_{n=1}^{\infty} a_n = 0.72$. But $a_{n+1} < a_n$ does not hold for all *n*.

   c. $\lim_{n\to\infty} s_n = 0.72$ where $s_n = a_1 + a_2 + \ldots + a_n$ TRUE. This is the definition of convergence of a series.

   d. $\lim_{n\to\infty} \frac{b_n}{c_n}$ converges. TRUE. $\lim_{n\to\infty} \frac{b_n}{c_n} = \lim_{n\to\infty} \frac{n^2}{(n+1)^3} = 0$ since the denominator has a polynomial with larger degree than the polynomial in the numerator.

   e. $\sum_{n=1}^{\infty} \frac{b_n}{c_n}$ converges. FALSE. $\sum_{n=1}^{\infty} \frac{n^2}{(n+1)^3} > \sum_{n=1}^{\infty} \frac{n^2}{(n)^3} = \sum_{n=1}^{\infty} \frac{1}{n}$. But $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (*p* = 1) and by comparison test so must $\sum_{n=1}^{\infty} \frac{b_n}{c_n}$.

   f. $\sum_{n=1}^{\infty} (-1)^n \frac{b_n}{c_n}$ converges. TRUE. This is an alternating series. $\lim_{n\to\infty} \frac{b_n}{c_n} = \lim_{n\to\infty} \frac{n^2}{(n+1)^3} = 0$ and $\frac{n^2}{(n+1)^3}$ decreases to zero as *n* increases. So by alternating series test, the series converges.