7. [12 points] Suppose that functions $f(x)$, $g(x)$, and $h(x)$ are continuous and differentiable for $x \geq 1$ and satisfy the condition that $0 \leq f(x) \leq g(x) \leq h(x)$ for $x \geq 1$. Furthermore, suppose that $\int_1^\infty g(x)dx$ converges.

You do not need to show your work for this page. No partial credit will be given.

a. [4 points] Consider the following group of statements:

I. $\int_1^\infty h(x)dx$ diverges.

II. $\int_1^\infty h(x)dx$ converges.

III. $\int_1^3 h(x)dx$ converges.

Which of the above statements must be true? Circle ONE of the following choices:

A. I is true.

B. II is true.

C. III is true.

D. I and III are true.

E. II and III are true.

b. [4 points] Consider the following group of statements:

I. $\int_1^\infty f(x)dx$ diverges.

II. $\int_1^\infty f(x)dx$ converges.

III. $\int_1^3 f(x)dx$ converges.

Which of the above statements must be true? Circle ONE of the following choices:

A. I is true.

B. II is true.

C. III is true.

D. I and III are true.

E. II and III are true.

c. [4 points] Consider the following group of statements:

I. $\int_1^\infty (f(x) + g(x))dx$ converges.

II. $\int_1^\infty (h(x) + g(x))dx$ converges.

III. $\int_1^\infty \frac{g(x)}{x}dx$ converges.

Which of the above statements must be true? Circle ONE of the following choices:

A. I is true.

B. II is true.

C. III is true.

D. I and III are true.

E. II and III are true.