4. [12 points] A bank account earns 2.5% annual interest compounded continuously. Continuous payments are made out of the account at a rate of \$15,000 per year for 18 years.

a. [4 points] Write a differential equation describing the balance $B = f(t)$, where t is in years satisfying $0 \leq t \leq 18$.

Solution:

$$\frac{dB}{dt} = .025B - 15,000 = .025(B - 600,000)$$

b. [4 points] Solve the differential equation you found in part (a) given an initial balance of B_0.

Solution:

$$\begin{aligned} \frac{dB}{dt} &= .025(B - 600,000) \\ \int \frac{dB}{(B - 600,000)} &= \int .025dt \\ \ln|B - 600,000| &= .025t + C \\ B - 600,000 &= Ae^{.025t} \\ B &= Ae^{.025t} + 600,000 \end{aligned}$$

Given $B = B_0$ when $t = 0$, we have $B_0 = A + 600,000$, so $A = B_0 - 600,000$, giving $B = (B_0 - 600,000)e^{.025t} + 600,000$.

c. [4 points] What was the initial balance if the account has \$10,000 remaining 18 years after the account was opened? Give your answer to the nearest penny.

Solution: Solving for B_0 given that $B = 10,000$ when $t = 18$, we have $10,000 = (B_0 - 600,000)e^{.025(18)} + 600,000$, giving $-590,000 = (B_0 - 600,000)e^{.45}$, which leads to $B_0 = -590,000e^{-.45} + 600,000 \approx 223,799.39$.
The initial balance would be approximately \$223,799.39.