4. [12 points] A bank account earns 2.5% annual interest compounded continuously. Continuous payments are made out of the account at a rate of $15,000 per year for 18 years.

a. [4 points] Write a differential equation describing the balance \( B = f(t) \), where \( t \) is in years satisfying \( 0 \leq t \leq 18 \).

**Solution:**
\[
\frac{dB}{dt} = 0.025B - 15,000 = 0.025(B - 600,000)
\]

b. [4 points] Solve the differential equation you found in part (a) given an initial balance of \( B_0 \).

**Solution:**
\[
\int \frac{dB}{B - 600,000} = \int 0.025 dt
\]
\[
\ln |B - 600,000| = 0.025t + C
\]
\[
B - 600,000 = Ae^{0.025t}
\]
\[
B = Ae^{0.025t} + 600,000
\]

Given \( B = B_0 \) when \( t = 0 \), we have \( B_0 = A + 600,000 \), so \( A = B_0 - 600,000 \), giving \( B = (B_0 - 600,000)e^{0.025t} + 600,000 \).

c. [4 points] What was the initial balance if the account has $10,000 remaining 18 years after the account was opened? Give your answer to the nearest penny.

**Solution:** Solving for \( B_0 \) given that \( B = 10,000 \) when \( t = 18 \), we have \( 10,000 = (B_0 - 600,000)e^{0.025(18)} + 600,000 \), giving
\[
-590,000 = (B_0 - 600,000)e^{-0.45}, \text{ which leads to } B_0 = -590,000e^{-0.45} + 600,000 \approx 223,799.39.
\]
The initial balance would be approximately $223,799.39.