7. [12 points] Suppose that functions \( f(x) \), \( g(x) \), and \( h(x) \) are continuous and differentiable for \( x \geq 1 \) and satisfy the condition that \( 0 \leq f(x) \leq g(x) \leq h(x) \) for \( x \geq 1 \). Furthermore, suppose that \( \int_1^\infty g(x)dx \) converges.

You do not need to show your work for this page. No partial credit will be given.

a. [4 points] Consider the following group of statements:

I. \( \int_1^\infty h(x)dx \) diverges.

II. \( \int_1^\infty h(x)dx \) converges.

III. \( \int_3^\infty h(x)dx \) converges.

Which of the above statements must be true? Circle ONE of the following choices:

A. I is true.
B. II is true.
C. III is true.
D. I and III are true.
E. II and III are true.

b. [4 points] Consider the following group of statements:

I. \( \int_1^\infty f(x)dx \) diverges.

II. \( \int_1^\infty f(x)dx \) converges.

III. \( \int_3^\infty f(x)dx \) converges.

Which of the above statements must be true? Circle ONE of the following choices:

A. I is true.
B. II is true.
C. III is true.
D. I and III are true.
E. II and III are true.

c. [4 points] Consider the following group of statements:

I. \( \int_1^\infty (f(x) + g(x))dx \) converges.

II. \( \int_1^\infty (h(x) + g(x))dx \) converges.

III. \( \int_1^\infty \frac{g(x)}{x}dx \) converges.

Which of the above statements must be true? Circle ONE of the following choices:

A. I is true.
B. II is true.
C. III is true.
D. I and III are true.
E. II and III are true.