2. [14 points] A microphone at the point \( r = 0 \) detects sounds in a region enclosed by the cardioid 
\[ r = 2 + \frac{15}{8} \cos \theta. \] 
The microphone is placed in front of the stage at an auditorium to record a musical band. Let \( d \) denote the smallest distance you must leave between the audience and the microphone to avoid recording any noise from the public in attendance.

\[ \begin{align*}
\text{Stage} & \\
\text{Audience} & \\
\text{Microphone} & \\
\end{align*} \]

\[ r = 2 + \frac{15}{8} \cos \theta \]

\[ (r_1, \theta_1) \quad (r_2, \theta_2) \]

\[ x = d \quad x = 0 \]

\[ A \]

a. [5 points] Write an integral that computes the area of the shaded region \( A \) in terms of \( \theta_1, \theta_2 \) and \( d \).

b. [4 points] Write a formula in terms of \( \theta \) that computes the value of the slope of the tangent line to the cardioid.

c. [3 points] Find an exact expression for the values of \( 0 \leq \theta < 2\pi \) at which the cardioid has a vertical tangent line. Full credit will not be given for decimal approximations.

d. [2 points] Find the value of \( d \). Show all your work.