2. [14 points] A microphone at the point r = 0 detects sounds in a region enclosed by the cardioid $r = 2 + \frac{15}{8} \cos \theta$. The microphone is placed in front of the stage at an auditorium to record a musical band. Let d denote the smallest distance you must leave between the audience and the microphone to avoid recording any noise from the public in attendance.



- **a**. [5 points] Write an integral that computes the area of the shaded region A in terms of θ_1 , θ_2 and d.
- **b.** [4 points] Write a formula in terms of θ that computes the value of the slope of the tangent line to the cardioid.

c. [3 points] Find an exact expression for the values of $0 \le \theta < 2\pi$ at which the cardioid has a vertical tangent line. Full credit will not be given for decimal approximations.

d. [2 points] Find the value of d. Show all your work.