

3. [14 points] A farmer notices that a population of grasshoppers is growing at undesirable levels in his crop. He decides to hire the services of a pest control company. They offer the farmer a pesticide capable of eliminating the grasshoppers at a rate of 1 thousand grasshoppers per week. In the absence of pesticides, it is estimated that the grasshopper population grows at a rate of 20 percent every week. Let  $P(t)$  be the number of grasshoppers (in thousands)  $t$  weeks after the pesticide is applied to the crop. Then  $P(t)$  satisfies

$$\frac{dP}{dt} = \frac{P}{5} - 1.$$

Suppose there are  $P_0$  thousand grasshoppers in the crop at the time the pesticide is applied in the crop.

- a. [8 points] Find a formula for  $P(t)$  in terms of  $t$  and  $P_0$ .

- b. [3 points] Does the differential equation have any equilibrium solutions? List each equilibrium solution and determine whether it is stable or unstable. **Justify your answer.**

- c. [3 points] Does the effectiveness of the pesticide depend on  $P_0$ ? That is, is the pesticide guaranteed to eliminate the grasshopper population regardless of the value of  $P_0$ , or are there some values of  $P_0$  for which the grasshoppers will survive? If so, determine these values of  $P_0$ .