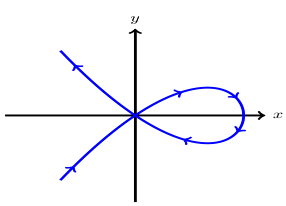
1. [14 points] Indicate if each of the following statements are true or false by circling the correct answer. **Justify your answers.**

a. [2 points] The function $z(t) = \sin(at) + at$ is a solution to the differential equation $z'' + a^2 z = a^3 t$.

$\boxed{\text{True}}$ False

Solution: $z' = a\cos(at) + a \quad z'' = -a^2 \sin(at)$
$z'' + a^2 z = a^2 \sin(at) + a^2(\sin(at) + at) = a^3 t$.

b. [3 points] The motion of a particle is given by the parametric curve $x = x(t)$, $y = y(t)$ for $0 \leq t \leq 3$ shown below. The arrows indicate the direction of the motion of the particle along the path. If the curve passes only twice through the origin, $x(1) = x(2) = 0$ and $y(1) = y(2) = 0$ then $\dfrac{\frac{d}{dt}\left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)}{\frac{dx}{dt}} > 0$ for $t = 1$.



True $\boxed{\text{False}}$

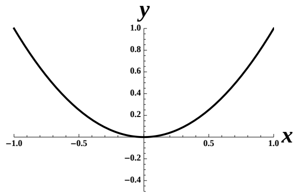
Solution: The first time that the curve passes through the origin at $t = 1$, the curve has negative concavity.

c. [3 points] Euler's method yields an overestimate for the solutions to the differential equation $\frac{dy}{dx} = 4x^3 + 2x + 1$.

True $\boxed{\text{False}}$

Solution: False, $y'' = 12x^2 + 2 > 0$ then Euler method is an underestimate since y is concave up.

d. [3 points] The graph of $x = x(t)$ and $y = y(t)$ for $0 \le t \le 2$ is given below. If $y'(1) = 0$, then it must be the case that $(x(1), y(1)) = (0, 0)$.



True $\boxed{\text{False}}$

Solution: False. The particle can stop at any point at time $t = 1$ in the parabola. Then $y'(1) = 0$ and $x'(1) = 0$ without necessarily be the case that $(x(1), y(1)) = (0, 0)$.

e. [3 points] If $\int_0^2 f(x)dx$ is an improper integral, then $\int_0^1 f(x)dx$ must also be an improper integral.

True $\boxed{\text{False}}$

Solution: False. If $f(x) = \frac{1}{2-x}$ then $\int_0^2 f(x)dx$ is an improper integral, but $\int_0^1 f(x)dx$ is not an improper integral.