2. [14 points] A microphone at the point \( r = 0 \) detects sounds in a region enclosed by the cardioid 
\( r = 2 + \frac{15}{8} \cos \theta \). The microphone is placed in front of the stage at an auditorium to record a
musical band. Let \( d \) denote the smallest distance you must leave between the audience and
the microphone to avoid recording any noise from the public in attendance.

a. [5 points] Write an integral that computes the area of the shaded region \( A \) in terms of
\( \theta_1, \theta_2 \) and \( d \).

\[
A = \int_{\theta_1}^{\theta_2} \frac{1}{2} \left( \frac{-d}{\cos \theta} \right)^2 d\theta - \int_{\theta_1}^{\theta_2} \frac{1}{2} \left( 2 + \frac{15}{8} \cos \theta \right)^2 d\theta
\]

b. [4 points] Write a formula in terms of \( \theta \) that computes the value of the slope of the
tangent line to the cardioid.

\[
\frac{dy}{dx} = \frac{y'}{x'} = \frac{(2 + \frac{15}{8} \cos \theta \sin \theta)' \sin \theta + (2 + \frac{15}{8} \cos \theta \cos \theta)' \cos \theta}{(2 + \frac{15}{8} \cos \theta \cos \theta)' \cos \theta - (2 + \frac{15}{8} \cos \theta \sin \theta) \sin \theta}
\]

c. [3 points] Find an exact expression for the values of \( 0 \leq \theta < 2\pi \) at which the cardioid
has a vertical tangent line. Full credit will not be given for decimal approximations.

\[
\text{Solution: } x' = (-\frac{15}{8} \sin \theta) \cos \theta - (2 + \frac{15}{8} \cos \theta) \sin \theta = -\cos \theta(2 + \frac{15}{8} \cos \theta) = 0.
\]
\[
\sin \theta = 0 \text{ then } \theta = 0, \pi.
\]
\[
2 + \frac{15}{8} \cos \theta = 0 \text{ then } \cos \theta = -\frac{8}{15}.
\]
This yields \( \theta = 0, \pi, \theta_1 = \cos^{-1}(-\frac{8}{15}), \theta_2 = 2\pi - \cos^{-1}(-\frac{8}{15}) \)

d. [2 points] Find the value of \( d \). Show all your work.

\[
\text{Solution: } d = -x(\theta_1) = -(2 + \frac{15}{8} \cos \theta_1) \cos \theta_1 = \frac{8}{15}
\]